Conference on Bluff Body Wakes and Vortex-Induced Vibrations (BBVIV3)
Port Douglas, Australia
17-20 December, 2002

Interactions of the wakes of two spheres placed side by side

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Abstract – The periodic coupled wakes of two spheres, placed side by side in a uniform stream at low Reynolds numbers, are studied experimentally and by Direct Numerical Simulations. Different regimes of interaction are observed, depending on the separation distance between the spheres. For touching spheres, a single wake is found. At small gap sizes, the two wakes interact strongly, and out-of phase shedding occurs. In-phase shedding is observed experimentally for intermediate gap sizes, whereas DNS does not reveal a preferred phase difference between the two wakes. At distances above three sphere diameters, shedding is uncorrelated. Variations of the critical Reynolds number and shedding frequencies with separation distance are shown.

Introduction – The flow around a sphere at low Reynolds numbers, and the transitions between different flow regimes occurring, as the flow velocity is increased, has attracted a renewed interest in recent years (Johnson & Patel 1999, Tomboulides & Orszag 2000, Ghidersa & Dušek 2000, Thompson *et al.* 2001, Schouveiler & Provansal 2002). Although the sphere is the simplest geometry representing a three-dimensional bluff body, a number of applications exist, ranging from the aerodynamics of sports balls to particle-laden or multi-phase flows, the latter two often involving quite low Reynolds numbers. When particle or bubble densities in these flows increase, the wakes of the immersed bodies start to influence each other.

The wake of a single sphere (diameter d) placed in a uniform flow (velocity U) undergoes a series of transitions, when increasing the Reynolds number $Re = Ud/\nu$ (ν: kinematic viscosity), which are now well documented. Up to $Re \approx 210$, the sphere wake is axisymmetric and exhibits a recirculation region in the form of a vortex ring. Above this value, axisymmetry is lost, and the wake takes the form of two counter-rotating streamwise vortices, which are stationary and present a planar symmetry. Time-dependence first comes in at $Re \approx 280$, when the wake starts to oscillate, accompanied by the periodic shedding of vortex loops, which still retain the planar symmetry of the previous stationary regime. This symmetry and the strict periodicity of the wake are in turn lost above $Re \approx 360$, when low frequency modulations and variations in the orientation of the plane of symmetry appear.

In this paper, we consider the interaction of the wakes of two stationary spheres, placed side by side in a uniform flow, for Reynolds numbers between 200 and 350, *i.e.*, in a range where, for the case of a single sphere, the transition from a stationary to a periodic wake occurs. As for the well-known case of wake interference from two circular cylinders (see, *e.g.*, Bearman & Wadcock 1973, Williamson 1985, Peschard & Le Gal 1996), the degree of interaction for two sphere wakes is expected to depend mainly on the separation distance between the spheres. Previous studies of related problems include those by Kim *et al.* (1993), for stationary flows up to $Re = 150$, and by Tsuji *et al.* (1982), with some results on wake interference from three spheres at $Re = 470$.

Technical details – The flow was studied both experimentally and by Direct Numerical Simulation (DNS). Experimental visualisations were obtained in a water channel of cross section 12 × 15 cm^2. The two Teflon spheres of diameter d = 10 mm were held from upstream by two rigid bent hollow rods (external diameter 1.5 mm), connected to an adjustable support outside the test section, which allowed variation of the distance h between the centres of the spheres. The part of each rod attached to the respective sphere was inclined by about 15° with respect to the direction of the free stream, in a plane perpendicular to the line joining the two sphere centres. Fluorescent dyes were injected through small downstream holes connected through a channel across the spheres with the hollow upstream support rods. The flow was illuminated by the light of a 5 W Argon ion laser and recorded on standard VHS video. Quantitative measurements of wake frequencies were obtained using the same system of spheres and supports in a low-speed low-turbulence wind tunnel with a 25 × 25 cm^2 cross section. The free stream velocity U was measured by Laser Doppler Anemometry in the section containing the sphere centres, thus accounting automatically for blockage and boundary layer growth. Wake spectra were obtained from single hot wire measurements. The two parameters governing the flow are the Reynolds number based on the diameter of one sphere and the non-dimensional separation distance h/d. The latter parameter was varied between 1 (touching spheres) and 4.6.

Direct Numerical Simulations of the two-sphere wake were performed for Re = 300 and different separation distances using a three-dimensional spectral-element code. Use of a high-order splitting method results in second-order temporal accuracy. The rectangular simulation domain extends from 4d upstream to 20d downstream, and has cross-section dimensions of 8d + h and 8d in the directions parallel and perpendicular to the line joining the two spheres, respectively. The computational mesh contains up to 1764 seventh-order spectral elements with up to 626,000 nodes, whose density is maximum in the vicinity of the sphere and in the near wake where velocity gradients are highest. Full details of the method, including extensive code validation, can be found in Brydon (2000).

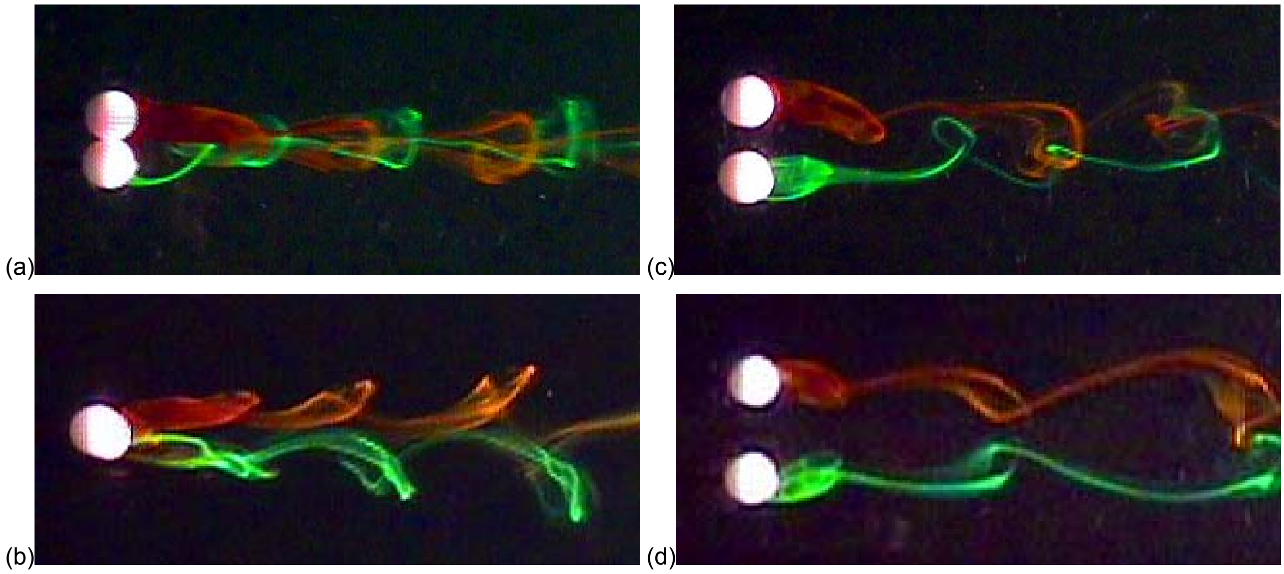


Figure 1. Dye visualisation of the coupled wakes of two spheres at Re = 330.
(a) and (b): h/d = 1.0, top and side view, respectively; (c) h/d = 1.6; (d) h/d = 1.9.

Results – For Reynolds numbers in the range corresponding to a periodic single-sphere wake, experimental flow visualisation (Figure 1) has shown the existence of the following regimes of wake interaction for two spheres side by side, as a function of their separation distance.

When the two spheres are touching (h/d = 1), they act as a single bluff body, whose wake exhibits a symmetry with respect to the plane separating the two spheres (Figure 1(a)). The side view visualisation in Figure 1(b) bears a strong resemblance with images of a two-dimensional Kármán vortex street behind a circular cylinder, with alternate shedding of dye/vorticity on both sides of the wake. This is in agreement with earlier observations by Tsuji *et al.* (1982) of the wake behind three touching spheres. As shown below, the shedding frequency of this “merged” two-sphere wake is significantly higher than in the other regimes with non-zero gaps between the spheres, and which are all characterized by the existence of two identifiable wakes, one behind each sphere.

The visualisation in Figure 1(c) represents the coupled wake pattern for separation distances h/d in the approximate range 1.2–1.7, *i.e.* for small gap sizes (less than one sphere diameter). Each sphere is observed to shed vortex loops individually, in a way similar to the periodic shedding found for a single sphere. The wake has an overall planar symmetry, but now with respect to the plane containing the two sphere centres, and the downstream ends of the vortex loops curve towards the wake centreline. For a single sphere, the orientation of the plane of symmetry of the periodic wake is arbitrary; it is determined by the initial conditions of the flow or (in experiments) by the presence of a symmetry-breaking support system. In the present case, the second sphere introduces a preferred direction for the orientation of the wake. Correspondingly, the lift force on each sphere is found to be oriented in a direction aligned with the plane of symmetry in the asymptotic wake, as shown by the numerical results in Figure 2(b). In this regime, shedding from the two spheres occurs with a phase shift of 180°. This out-of-phase shedding is well illustrated by the temporal evolution of the drag coefficients for both spheres, obtained from numerical simulation and shown in Figure 2(a). The proximity of the two spheres in this regime leads to a mutual deformation of their respective wakes. Figure 1(c) shows that the dye filaments connecting two successive heads of the vortex loops shed from one sphere are distorted by the flow induced by the loop from the other sphere’s wake. For a single sphere, the region between two loops, which in experimental dye visualisations only shows as a pair of threads, actually also contains vortical structures in the form of induced hairpin vortices. The existence of this “phantom” vorticity was first shown by Johnson & Patel (1999), and is also found in the present study (see Figure 3). The interaction between the induced vortices of one sphere wake with the primary loop from the other appears to be responsible for the strong deformations, leading to structures that have a streamwise wavelength of half the one initially found in each separate wake.

For intermediate spacings of the sphere, *i.e.* for 1.7 < h/d < 3 approximately, the interaction between the two wakes is much weaker. The wake of each sphere is found to exhibit a downstream evolution very similar to the one for an isolated single sphere, as illustrated in the visualisation in Figure 1(d). However, a coupling still exists: both wakes are again symmetric with respect to the plane containing the sphere centres, but now the periodic shedding of vortex loops occurs simultaneously, leading to an additional planar symmetry of the overall wake with respect to the plane between the spheres. In the experiments, this in-phase shedding was systematically observed in this range of h/d. In the numerical simulations, other phase angles between the two wakes were also observed for h/d = 2 (~180° phase shift) and h/d = 2.5 (~90° phase shift), where the flow was started from rest and from an initial condition built from two single-sphere wakes, respectively. At present, it is not clear if different metastable states exist for the coupled sphere wakes in this parameter range, or if the presence of the support structures somehow selects the in-phase shedding in the experiments, whereas in the idealised flow the phase difference might be more or less random. This point is under investigation.

For separation distances larger than about three sphere diameters, both experiment and DNS show no preferred phase difference of the shedding from the two spheres. In the DNS, a very weak coupling is nevertheless

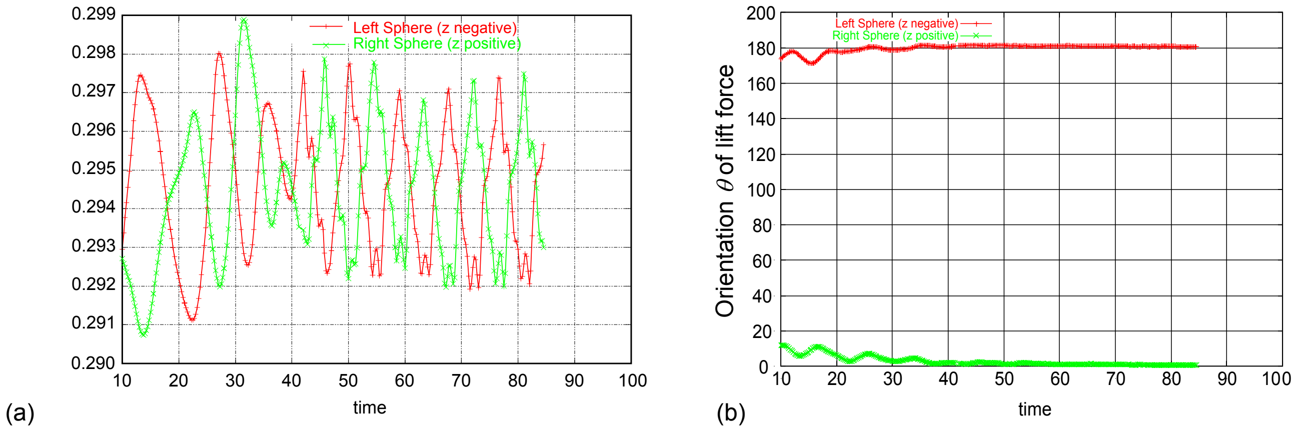


Figure 2. Time dependent drag coefficients (a) and directions of the lift force (b) for the two spheres, obtained numerically for Re = 300 and h/d = 1.5. θ = 0 and θ = 180° are parallel to the line joining the spheres, the time unit is D/U.

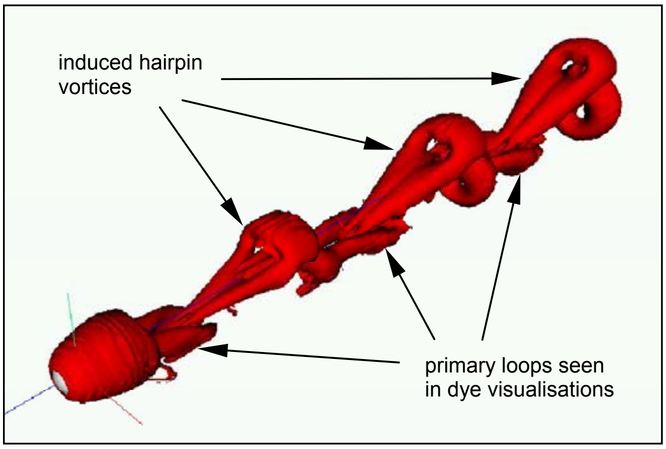


Figure 3. Perspective view of vortical structures in the wake of a single sphere at Re = 300, visualised using the method proposed by Jeong & Hussain (1995).

still observed at h/d = 3.5, leading to a slow drift of the orientation of the plane of symmetry for each separate wake towards the line going through both sphere centres, similar to the situation observed for the previous regimes with stronger coupling.

Quantitative results characterizing the different regimes are given in Figures 4 and 5. The critical Reynolds number for the onset of time-dependent periodic wake flow is shown in Figure 4 as function of the separation distance. For touching spheres, it is found to be Re_c = 219. This case may be compared to the flow around a short cylinder with free hemispherical ends and aspect ratio 2, investigated by Schouveiler & Provansal (2001). They find a significantly lower value of Re_c = 155. As soon as the spheres are moved apart, the critical Re increases rapidly within the regime of strong coupling and out-of-phase shedding, to reach a value close to 280. From thereon, the variation slows down, it gradually approaches the value of Re_c = 283 measured for a single sphere with the present experimental set-up.

The shedding frequencies of the coupled wakes are plotted in Figure 5 in the form of Strouhal numbers $St = fd/U$ (f: wake frequency). Figure 5(a) shows the experimentally observed frequency variations in the periodic regimes for various values of h/d. The different onset Reynolds numbers are clearly seen, as well as an interesting hysteresis phenomenon occurring for the case of touching spheres around Re = 280. For a fixed Reynolds number, the Strouhal numbers are found to vary smoothly with h/d for non-vanishing gaps (Figure 5(b)). It increases in the regime of strong coupling, reaching a maximum just under h/d = 2, from where it then decreases again, asymptoting the value for a single sphere at large separations. The DNS values, although consistently lower than the experimental measurements, show the same overall trend. The Strouhal number obtained numerically for a single sphere (St = 0.136) is in excellent agreement with other recent studies (Johnson & Patel 1999, Tomboulides & Orszag 2001). The case of touching spheres represents an exception to the general behaviour. The wake frequency is much higher (St = 0.224) than for the other regimes, and closer to the one for a circular cylinder of diameter d. It is interesting to note that for a cylinder of aspect ratio 2 with free hemispherical ends, the regime of periodic single-frequency shedding only extends up to $Re \approx 190$ (Schouveiler & Provansal 2001), whereas the wake of the two touching spheres (*a priori* a rather similar geometry) is periodic up to $Re \approx 370$. Unfortunately, no reliable data could be obtained in the experiments for the range $1 < h/d < 1.3$, due to the presence of strong vortex-induced mechanical vibrations of the sphere-support systems at these very small gaps. The characterization of the transition between a single wake and two strongly coupled wakes occurring in this interval will be the object of further investigations.

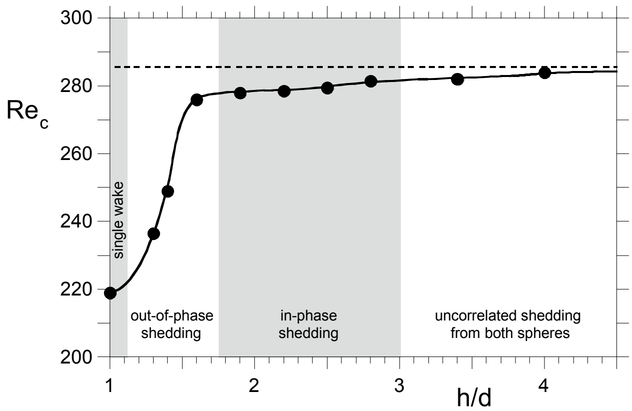
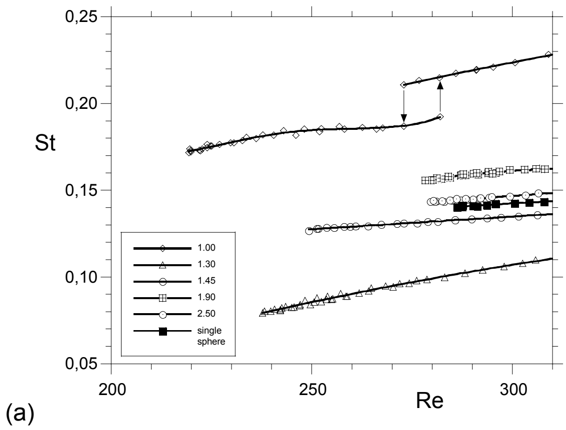


Figure 4. Critical Reynolds number for the onset of time-dependent (periodic) wake structure, as a function of separation distance (from experiment). The dotted line represents the value measured for a single sphere.



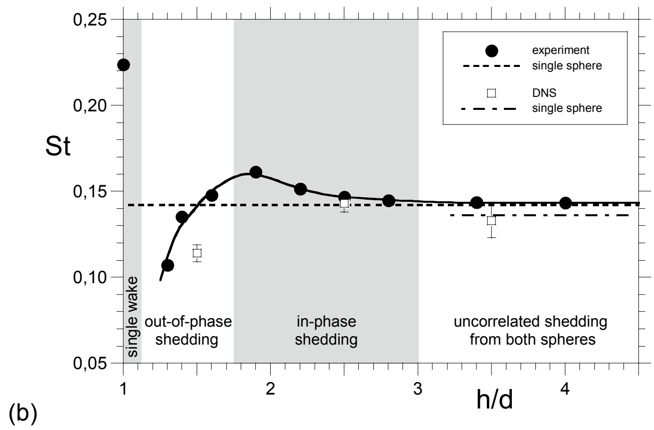


Figure 5. Strouhal numbers of the coupled sphere wakes; (a) as a function of Reynolds number for different separation distances (experiment); (b) as a function of separation distance at *Re* = 300 (experiment and DNS).

Conclusions – Experiments and Direct Numerical Simulations of the wakes of two stationary spheres placed side by side in a uniform flow at low Reynolds numbers, have revealed the existence of different regimes of interaction, as a function of the spheres' separation distance. When increasing this distance from one diameter (touching spheres), one successively observes a single wake, two coupled wakes with out-of-phase shedding, two wakes with in-phase shedding, and independent shedding from both spheres. The critical Reynolds number for the onset of shedding decreases rapidly with decreasing separation distance, once the latter drops below two sphere diameters. At a given Reynolds number, the wake frequency varies smoothly with separation distance in the range where two separate wakes can be identified. As the spheres are brought closer together, it first increases above, then decreases below the value for an isolated sphere. For touching spheres, the frequency of oscillation is closer to that for a circular cylinder, and a hysteresis in the Strouhal-Reynolds number relationship is found around *Re* = 280. Good agreement exists between experimental and numerical results.

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