Longitudinal vortex structures in a cylinder wake

J. Wu
Division of Building, Construction and Engineering, CSIRO, Highett 3190, Australia

J. Sheridan
Department of Mechanical Engineering, Monash University, Clayton 3168, Australia

M. C. Welsh, K. Hourigan, and M. Thompson
Division of Building, Construction and Engineering, CSIRO, Highett 3190, Australia

(Received 28 February 1994; accepted 29 March 1994)

This paper presents the velocity field of the longitudinal vortices found in the wake of a circular cylinder, as measured using digital particle image velocimetry (PIV). Vorticity and circulation of the longitudinal vortices are presented, based on instantaneous velocity distributions in a transverse plane behind the cylinder.

Recently, flow visualizations by Wei and Smith,[1] Williamson,[2] Welsh *et al.*,[3] and Bays-Muchmore and Ahmed[4] have shown that three-dimensional (3-D) vortical structures develop in the wake of a bluff body. The 3-D vortical structures were found to include pairs of counter-rotating longitudinal vortices superimposed on the nominally two-dimensional (2-D) Kármán vortex street as speculated by Grant[5] decades ago. The vortices are similar to the 3-D flow structures observed by Bernal and Roshko[6] in plane mixing layers. While the existence and the topological details of the longitudinal vortex structures have been recognized through visual observation, the physical characteristics and dynamical properties of the structures remain to be explored through quantitative measurement.

In the present study, digital particle image velocimetry (PIV) has been used to measure the instantaneous velocity field in a transverse plane (the x-z plane, where the x axis is in the flow direction and the z axis is parallel to the cylinder axis) behind the circular cylinder. The principle of PIV is simple: the flow field, seeded with particles, is illuminated by a pulsed laser light sheet, the displacements of the double or multiply exposed particles are recorded and then analyzed using specially developed software to obtain instantaneous velocity distributions. PIV techniques have attracted substantial attention in recent years and comprehensive reviews have been provided by Adrian[7] and Buchhave.[8] In this application, single frame and multiply exposed digital particle images were acquired using a CCD camera with a spatial resolution of 1280×1024 pixel. The experiment was carried out in a low turbulence water tunnel at a Reynolds number of 525 based on cylinder diameter. This Reynolds number was chosen to be well above the transitional range noted by Williamson[2] for the onset of the longitudinal vortices, so that the structures were expected to be fully developed and representative of a wider range of Reynolds number. The measurements were conducted in a transverse plane 2D behind the back of the cylinder and covered an area of approximately $2.8\times2.2D$. The in-plane velocity vectors, with approximately 2000 points per frame, were obtained using Young's fringe patterns, calculated from 2-D FFT over small interrogation windows overlapped at 50%. The overall velocity measurement uncertainty is estimated to be 4% and the vorticity error 15%~20%, at the 95% confidence level.

A typical hydrogen-bubble flow visualization pattern is presented in Fig. 1(a). It can be seen that mushroom-type structures, indicated by the arrow, develop in the near wake behind a circular cylinder, implying the existence of the counter-rotating longitudinal vortices in the cylinder wake. An instantaneous velocity distribution sampled at an arbitrary instant is presented in Figs. 1(b) and 1(c), with the frame of reference moving at a speed approximately equal to the eddy convection velocity $U_e=60\%\ U_0$, where U_0 is the free-stream velocity. The cross-sectional streamline patterns were obtained by integrating the velocity field. The spiraling of the streamline patterns near vortex centers is indicative of flow three-dimensionalities, i.e., flow motions out of the measurement plane. From a survey of over 130 vortex pairs contained in 50 instantaneous velocity data frames, it was found that the streamlines usually *spiral in* around vortex centers. This suggests that the vortices are experiencing expansive strain fields perpendicular to the measurement plane.

Transverse vorticity has been calculated from the discrete velocity data. Figure 2 shows fluctuations of the maximum and minimum transverse vorticity component of vortex pairs as the test is repeated, where free-stream velocity and cylinder diameter are used for normalization: $\xi_y=\omega_y/(U_0/D)$, where ω_y is the transverse vorticity component. The mean of both positive and negative vorticity is approximately equal, $\xi_y\approx\pm7.3$. Measurements made using the same PIV technique show that the maximum spanwise vorticity of a shed Strouhal vortex is approximately $\xi_y=4\sim5$ at Re=525. Therefore the maximum vorticity of the longitudinal vortices is greater than that of the spanwise vortices.

This result is perhaps expected, as longitudinal vortices are thought to be stretched by spanwise vortices, as suggested by Wei and Smith[1] and Meiburg and Lasheras,[9] among others. If the longitudinal vortices are hypothesized to originate from the spanwise vortices, then a higher vorticity of the longitudinal vortices is consistent with the vortex stretching theory.

The circulation of a longitudinal vortex has been estimated by integrating vorticity over the area in which ξ_y is greater than 10% of $\xi_{y\ \max}$. The mean circulation of longitudinal vortices was found to be $\Gamma/(U_0D)=0.39$ with a standard deviation of 0.14, and the mean circulation of Kármán vortices was found to be $\Gamma_{KM}/(U_0D)=3.5$ with a standard

 1070-6631/94/6(9)/2883/3/$6.00

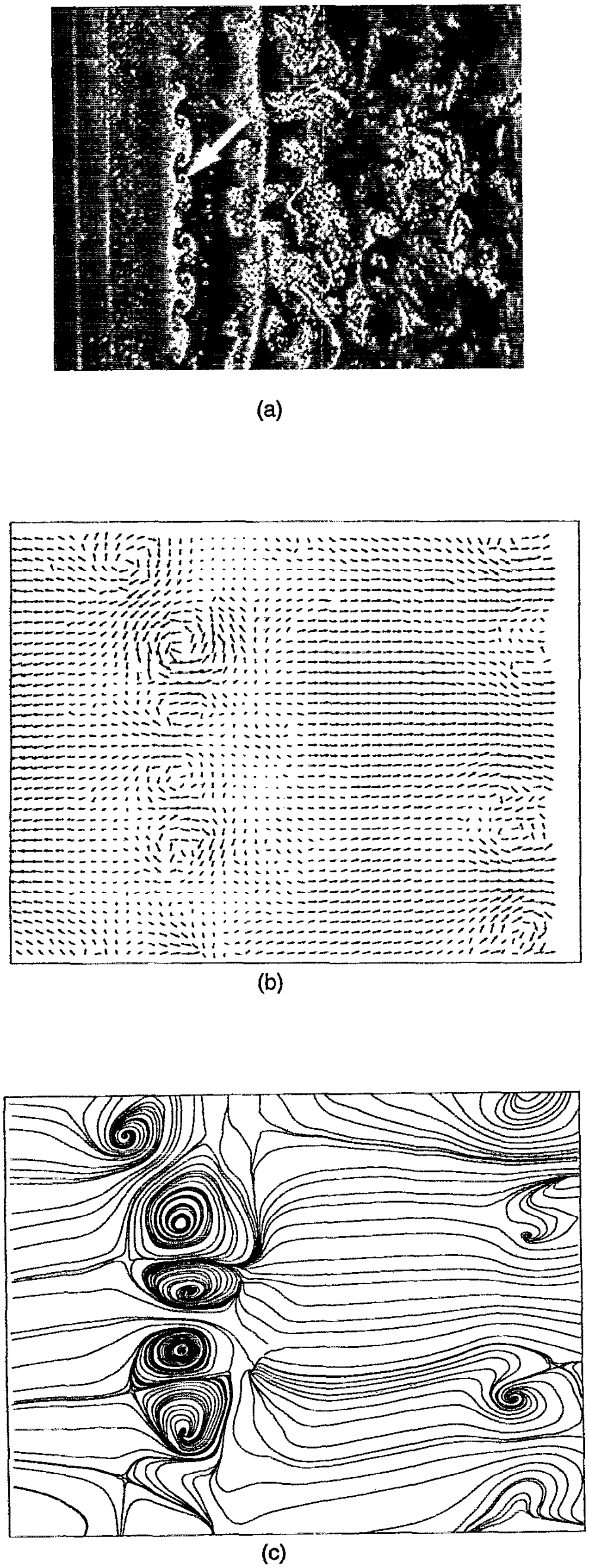

FIG. 1. Flow field in the transverse plane (the x-z plane): (a) flow visualization, flow from left to right and the cylinder is located at left; (b) velocity vectors seen in a frame of reference moving at 60% U_0; (c) sectional streamline seen in the moving frame of reference.

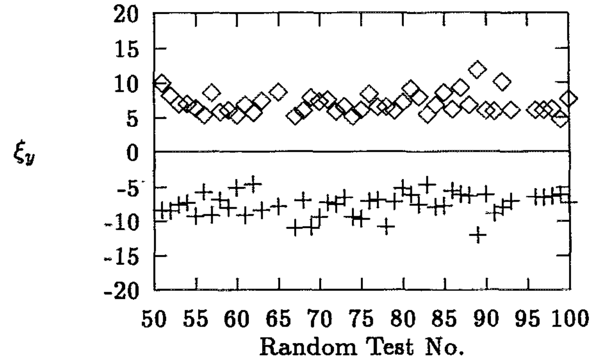

FIG. 2. Maximum (or minimum) vorticity fluctuations of longitudinal vortex pairs.

deviation of 0.27 (where $_{KM}$ denotes Kármán vortices). Thus the circulation of the longitudinal vortices is significantly smaller than that of the Kármán vortices:

$$\Gamma/\Gamma_{KM} \approx 0.11. \tag{1}$$

This suggests that the longitudinal vortices originate from only *part* of the spanwise vortices. This can be compared with the circulation of longitudinal vortices in mixing layers which has been given by Jimenez *et al.*[10] They investigated the three-dimensional topology of the streamwise vortices in a plane mixing layer, and they suggested a circulation ratio of 0.60. This value is larger than that found in the wake as presented here, and appears to indicate a difference in flow characteristics (between wakes and mixing layers).

To examine the symmetry of vortex pairs, the two circulations (positive and negative) of a vortex pair are plotted against each other in Fig. 3. A vortex pair is said to be ideal if its positive and negative circulation are equal, as indicated by the line. The data points, although showing a degree of scatter, clearly indicate a linear correlation. This correlation suggests that an increase in the circulation of one vortex corresponds to an increase (in absolute value) in the circula-

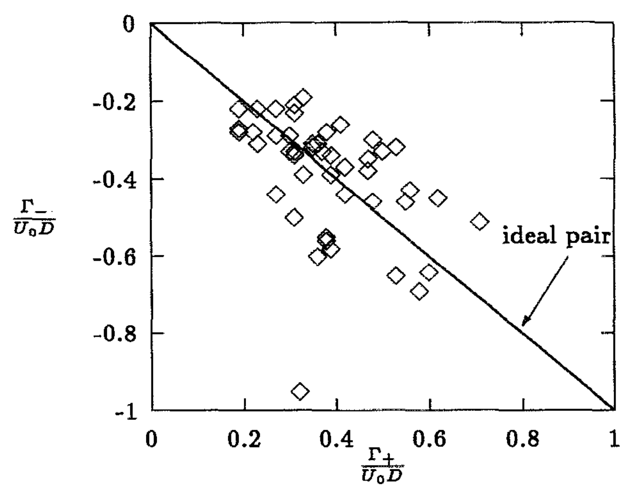

FIG. 3. A correlation between positive and negative circulations of vortex pairs.

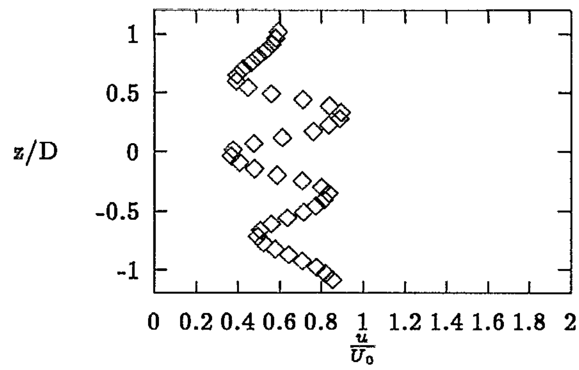

FIG. 4. Spanwise variation of u: an instantaneous velocity profile sliced through centers of longitudinal vortices.

tion of the other vortex of a counter-rotating vortex pair. The result adds useful quantitative support to the theory that the mushroom-type structures are actually part of a vortex loop,[9] since circulation is invariant along a vortex tube, even when it is being distorted.

To show the spanwise velocity variation caused by the existence of the longitudinal vortices, a typical instantaneous velocity profile through the centers of longitudinal vortices is shown in Fig. 4, where u is streamwise velocity and z is the spanwise axis (parallel to the cylinder axis).

In summary, we have presented the measured velocity field in the 3-D wake of a circular cylinder using the PIV method. Velocity vectors in the transverse plane behind the cylinder show a pattern of longitudinal vortices. Vorticity and circulation of the longitudinal vortices, which are useful in characterizing the vortical structures, have also been presented.

ACKNOWLEDGMENTS

The authors acknowledge the support from an Australia Research Council grant. J. Wu acknowledges the support from a Monash University Graduate Scholarship.

[1]T. Wei and C. R. Smith, "Secondary vortices in the wake of circular cylinders," J. Fluid Mech. **169**, 513 (1986).

[2]C. H. K. Williamson, "The existence of two stages in the transition to three-dimensionality of a cylinder wake," Phys. Fluids **31**, 3165 (1988).

[3]M. C. Welsh, J. Soria, J. Sheridan, J. Wu, K. Hourigan, and N. Hamilton, "Three-dimensional flows in the wake of a circular cylinder," *Album of Visualization* (The Visualization Society of Japan, Tokyo, 1992), No. 9, pp. 17–18.

[4]B. Bays-Muchmore and A. Ahmed, "On streamwise vortices in a turbulent wakes of cylinders," Phys. Fluids A **5**, 387 (1993).

[5]M. L. Grant, "The large eddies of turbulent motion," J. Fluid Mech. **4**, 149 (1958).

[6]L. P. Bernal and A. Roshko, "Streamwise vortex structure in plane mixing layers," J. Fluid Mech. **170**, 499 (1986).

[7]R. J. Adrian, "Particle imaging techniques for experimental fluid mechanics," Annu. Rev. Fluid Mech. **23**, 261 (1991).

[8]P. Buchhave, "Particle image velocimetry—status and trends," Exp. Thermal Fluid Sci. **5**, 586 (1992).

[9]E. Meiburg and J. C. Lasheras, "Experimental and numerical investigation of the three-dimensional transition in plane wakes," J. Fluid Mech. **190**, 1 (1988).

[10]J. Jimenez, M. Cogollos, and L. P. Bernal, "A perspective view of the plane mixing layer," J. Fluid Mech. **152**, 125 (1985).