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Sound Generated by Separated Flows Around Bluff Bodies

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ABSTRACT

In 1975 Howe published a theory of aerodynamic sound which described the interaction between flow and sound fields in terms of the vorticity and the acoustic particle velocities associated with the excited sound field. In this paper four published experimental studies of flow induced sound are interpreted in the light of this theory. The regions of flow where energy is transferred to the sound field are identified; the power is shown to depend on the phase of the sound cycle at which sections of vortical flow pass through these regions.

INTRODUCTION

Although flow excited sound has been studied for many years (Strouhal, 1878), it is only since Howe (1975) published his theory of aerodynamic sound that research workers have studied flow induced sound in terms of the fluid dynamics of the process; this involves an interaction between the vorticity in the flows and the acoustic particle velocities.

There are a number of examples showing this interaction process involves the shedding of discrete vortices synchronized with the sound frequency. Nelson, Halliwell and Doak (1981, 1983) studied the excitation of a Helmholtz resonator and found that the process was associated with the growth of a vortex from the cavity edge where the shear layer separated. The vortex then travelled across the opening and out of the resonator near the far edge. Nomoto and Culick (1982) made an analogous study of the excitation of loud acoustic resonances in a combustion model, which consisted of a flow duct containing two pairs of baffles displaced along the duct in the flow direction. They found that the excitation process was associated with shedding of vortices from the upstream pair of baffles which then passed through the throat between the downstream pair of baffles. The excitation of acoustic resonances in a duct containing a single plate shedding vortices from the trailing edge was described by Welsh, Stokes and Parker (1984), while the case of a single plate shedding vortices from its leading edge was described by Stokes and Welsh (1986). In both cases, the excitation process was associated with the shedding of vortices which passed through the sound fields surrounding the plates. Keller and Escudier (1983) examined the excitation of resonances in covered cavities and again found that the process was associated with vortices shedding from an edge. The synchronous shedding observed in these examples originates from the "feeding back" of sound onto the shear layers from which the vortices formed.

The aim of this paper is to show that the theory of aerodynamic sound due to Howe (1975) can be used to explain the fluid mechanics of the excitation process for all the flows in enclosed spaces referred to above. It describes the mechanism by which energy is transferred from the flow to sustain the resonance.

ACOUSTIC MODES IN DUCTS OR CAVITIES

For a duct containing a centrally located plate, the simplest transverse acoustic mode is the Parker β-mode (Parker 1966). This is a standing wave in which the acoustic velocities are always in the directions indicated in Figure 1, but with harmonically oscillating magnitudes, so that the sense alters every half-cycle. The nodal surface of zero acoustic pressure variation is the horizontal mid-surface. For the baffles in the flow duct described by Nomoto and Culick (1982), the simplest resonant acoustic mode is shown in Figure 2. In this case the acoustic pressure node is the vertical plane midway between the baffles, and the acoustic particle velocities are in a generally axial direction. The simplest acoustic mode associated with the Helmholtz resonator is shown in Figure 3. The acoustic particle velocities oscillate back and forth through the neck of the resonator.

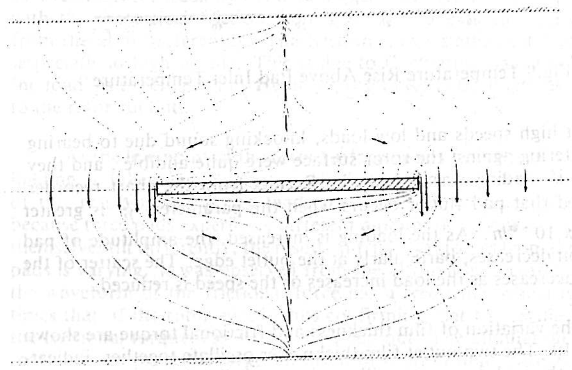

Fig 1: β-mode acoustic field for a plate in a duct; acoustic isobars (- - -); acoustic velocity direction, (———).

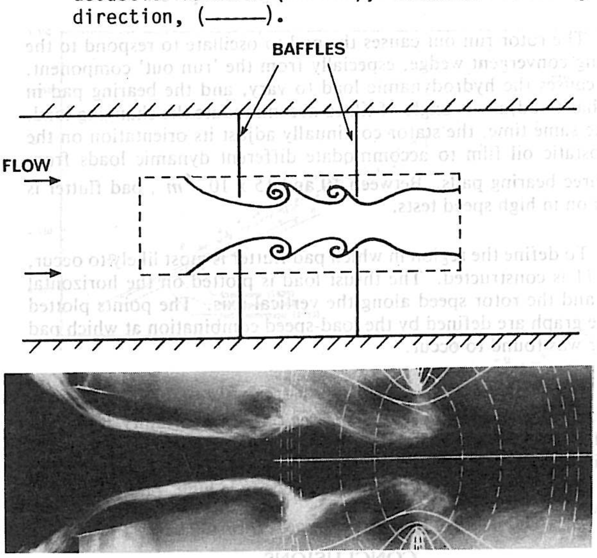

Fig 2: Flow visualization of locked vortex shedding during resonance in a duct containing two pairs of baffles (the first pair appears obliquely in the photograph, because of camera perspective); acoustic isobars, (- - -); direction of acoustic particle velocities, (————), direction of acoustic particle velocities;→ acoustic particle velocities.

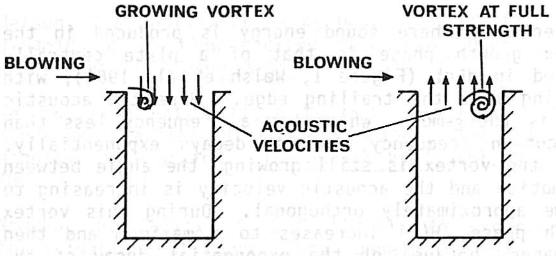

Fig 3: Diagrams of vortex shedding in a Helmholtz resonator, one vortex is shed each cycle and reaches the other edge a cycle later.

THE EXCITATION OF SOUND BY VORTICITY; HOWE'S THEORY OF AERODYNAMIC SOUND

For a single vortex passing through a sound field with a velocity $\underline{v}$, Howe (1975) showed that the rate P at which the vortex does work on the sound field is given by:-

$$P = k\,\underline{\omega} \cdot (\underline{v} \times \underline{u}) \qquad (1)$$

where $\underline{\omega}$, $\underline{v}$ and $\underline{u}$ are the vorticity, vortex velocity and the acoustic particle velocity vectors respectively. k is proportional to the air density and the length of the vortex tube; the density and the length are constant in the flows described here, so k is also a constant.

When the sound field is a standing wave, the acoustic velocity is the product of a spatial field $\underline{u_0}$ and a sinusoidal oscillation $\sin(2\pi ft + h)$, where h is a phase constant. It is useful to factorise P:

$$P = kH(t)\sin(2\pi ft + h) \qquad (2)$$

$$\text{where } H(t) = |\underline{u_0}|\,|\underline{v}|\,|\underline{\omega}|\sin\alpha \qquad (3)$$

Here, α is the angle between $\underline{u_0}$ and it is assumed both are orthogonal to $\underline{\omega}$ for the two-dimensional flows considered here.

Figure 4 shows a trajectory of a vortex of small strength approaching a plate. The lengths of the lines shown on each side of the trajectory indicate the instantaneous values of P. Those above the vortex path signify power transferred to the sound field. In the relatively undisturbed flow upstream and downstream of the plate $\underline{u_0}$ are approximately constant; H(t) is therefore constant and P oscillates with constant amplitude. Near the plate this is no longer true; H(t) first increases, then becomes quite small as the vortex approaches the acoustic stagnation point near the middle of the plate. Figure 5 shows, on a logarithmic scale, H(t) and its three factors which vary (equation (3)). On the logarithmic scale, the effects of the factors are additive and of the three factors, by far the most important near the plate is $|\underline{u_0}|$.

The total energy contributed by the vortex during its life is obtained by integrating P from time zero (when the vorticity was generated) to infinity. This is a Fourier integral of the amplitude function H(t). A substantial value of P requires that H(t) have variations with rate of change comparable to the sound velocity sinusoid.

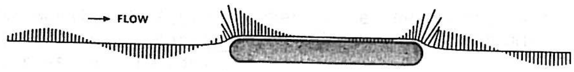

Fig 4: Instantaneous contributions of power transferred from a vortex to a resonant sound field; lines upward represent positive power; (——), vortex trajectory.

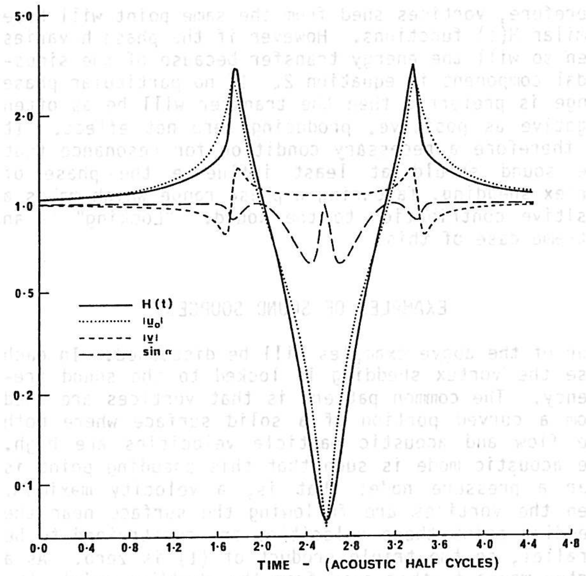

Fig 5: H(t), $|\underline{u_0}|$, $|\underline{v}|$ and sin α shown for a vortex following the same trajectory as in Figure 4. The peaks in H(t) occur when the vortex passes the ends of the plate.

Integrating P(t) numerically is difficult because of the oscillations; in the simplified example of Figure 4 it is impossible, because the oscillations do not diminish. In real cases the sound field eventually decays. It is much better to integrate by parts, because of the slow variation of H(t). The integrand is then:

$$Q = H'(t)\cos(2\pi ft + h)/2\pi f \qquad (4)$$

where H'(t) is the time derivative of H(t). The total energy contributed by the vortex during its life is the integral over time of Q (which is the same as that of P). Q is plotted in Figure 6. It still has an oscillatory component, but is very small where the flow is undisturbed, because H'(t) is small there.

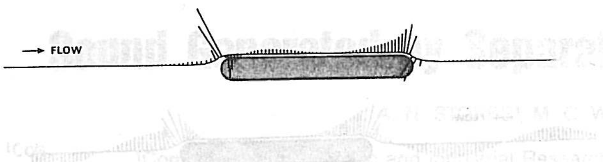

Fig 6: The modified integrand Q(t) shown for the same vortex trajectory as in Figure 4.

So far we have considered one vortex only. When a large number of vortices are shed consecutively from a single shedding point, their contributions need to be added. If the shedding is "locked", i.e. exactly periodic with the sound frequency, then all contributions are identical for a sound field that is stable over time. However if the shedding is not locked, the phase h may differ for consecutive vortices. Fortunately the factors making up H(t) have little dependence on the phase of the sound field. It is true that the vortex velocity is somewhat influenced by the sound, but the component which is induced is in the direction of the acoustic velocity, so does not appear in the vector product.

Therefore, vortices shed from the same point will have similar H(t) functions. However if the phase h varies then so will the energy transfer because of the sinusoidal component in equation 2. If no particular phase range is preferred then the transfer will be as often negative as positive, producing zero net effect. It is therefore a necessary condition for resonance that the sound should at least influence the phase of vortex shedding, favouring a phase range which makes a positive contribution to the sound. "Locking" is an extreme case of this.

EXAMPLES OF SOUND SOURCES

Four of the above examples will be discussed. In each case the vortex shedding is locked to the sound frequency. The common pattern is that vortices are shed from a curved portion of a solid surface where both the flow and acoustic particle velocities are high. The acoustic mode is such that this shedding point is near a pressure node; that is, a velocity maximum. When the vortices are following the surface near the shedding point these velocities are constrained to be parallel, so the triple product of (1) is zero. As a vortex moves further away from the shedding point, the angle α between the vortex velocity and the acoustic particle velocity increases and the vortex grows; the power H(t) potentially available increases at first but then diminishes with the decay in amplitude of the sound velocity.

Obstacles may be encountered later in the flow, and there H(t) may change rapidly. Whether a net source of sound results depends on the phase of the sound cycle at which the obstacle is encountered. Eventually, on further passage through and perhaps beyond the resonant cavity, the interaction between the vortex and sound field diminishes, either because of diffusion and cancellation of vorticity, or because the sound field diminishes in intensity. This diminution is too gradual to be a significant source of sound.

In the Helmholtz resonator (Figure 3) studied by Nelson, Halliwell and Doak (1981, 1983), the resonance is excited when air is blown across the slot in the resonator. Once in each cycle a vortex grows from the upstream edge, starting at a time when the acoustic velocities are directed into the resonator and the vortex is absorbing acoustic energy. The vortex grows as it moves across the throat of the resonator. By the time it approaches the far edge, the acoustic velocities have changed sign and the vortex is now doing work on the sound field. Being larger, it will generate more acoustic power than it absorbed in the previous half-cycle and there will be a net supply of energy to excite the resonator.

Another case where sound energy is produced in the vortex growth phase is that of a plate centrally located in duct (Figure 1, Welsh et al. 1984), with shedding from the trailing edge. Here the acoustic mode is the β-mode, which has a frequency less than the cut-on frequency, and so decays exponentially. While the vortex is still growing, the angle between its motion and the acoustic velocity is increasing to become approximately orthogonal. During this vortex growth phase, H(t) increases to a maximum and then decreases, because of the exponential decay of the sound field. If the maximum of H(t) occurs when the sound velocity is in the right direction, then the contribution to the sound energy during that half-cycle will outweigh the opposing contributions due to the neighboring half-cycles; a net source will result.

In the remaining two examples, the dominant sound source is remote from the point of vortex formation; it is in a region where the motion of the vortices is perturbed. If the leading edge of the plate previously described is made square, then shedding occurs from the leading edge (Stokes and Welsh 1986). There is some contribution to the energy integral as the growing vortices move out into the flow, but the sound velocity soon diminishes as the vortex moves along the plate surface. At the trailing edge, however, the sound and flow velocities increase again, the angle between them also increases, and more vorticity is entrained. This can produce a powerful acoustic source, provided the vortex passes at the appropriate part of the sound cycle. The shedding is locked, so this phase relation is determined at the shedding point; the phase at passage past the end therefore depends on the plate length and flow velocity. Typically there are distinct ranges of these variables during which resonance is possible, corresponding to different numbers of sound cycles which elapse while the vortex traverses the plate.

The last example is the combustion chamber model of Nomoto and Culick (1982) (Figure 2). The mode here is longitudinal rather than transverse. In locked shedding, vortices of opposite sign are shed simultaneously from opposite leading baffles and are deflected as they pass the trailing baffles. As in the previous examples, a vortex undergoing deflection is a powerful but phase dependent source of sound; the phase depends on the flow velocity and distance from leading to trailing baffles, and various ranges of these quantities allow resonance. In fact, as the vortex passes between the baffles, H(t) passes through zero because the vortex moves in the direction of the acoustic particle velocity. The phase which favours a source is one that ensures that u reverses its direction at the same time; the product then has the same sign on either side of the throat.

CONCLUSION

The Howe energy integral, applied to the time course of vortices released into a flow with acoustic resonance, provides a useful way of locating sound source regions, interpreting the conditions under which resonance is observed, and perhaps making it possible to influence these conditions by design.

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