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Stability of Wake Flows from Elongated Bluff Bodies

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ABSTRACT

The results from *Floquet stability analysis* of the flow around nominally two-dimensional long plates with elliptical leading edges and blunt trailing edges are presented, elucidating the early stages of wake transition to turbulent flow. Three modes of instability are found: Mode A, Mode B and Mode S. The first two of these also occur in wake transition for circular cylinders and square cross-sectioned cylinders. For sufficiently large aspect ratio, Mode S is found to be the dominant instability mode. This research indicates the generic turbulence transition scenario suggested for circular two-dimensional bodies does not apply to all two-dimensional bluff bodies.

1. INTRODUCTION

The generation of streamwise vortex structures in the wake of a nominally two-dimensional bluff body has been the subject of intense study and debate over the past fifty years [Roshko 1955, Gerrard 1966, Williamson 1996]. Over the last 20 years, progress in numerical and experimental technology has allowed researchers to accurately map both the parameter space governing the inception of these streamwise wake vortices and to explore the general geometry and dynamics of these streamwise wake vortices [Williamson 1996, Barkley and Henderson 1996, Henderson 1997]. It is generally accepted that the inception of streamwise vortex structures in the wake is the first step in the progression to a fully turbulent wake. Indeed, the overall wake structures observed in flow fields of relatively low Reynolds number have been shown to persist in flow fields having much higher Reynolds numbers [Williamson 1996]. Research on this topic has been inspired largely in an attempt to discover underlying features leading to fully turbulent flow and therefore to define progressively more accurate models to describe turbulent flow fields.

In an experimental study, Williamson (1996) identified 2 discontinuous alterations of the flow field in the wake of a circular cylinder as the flow changes from a two- to three-dimensional state. These changes consist of a periodic distortion of the Karman vortex-street in the cylinder spanwise direction and the generation of streamwise vortex structures in the wake of the flow field. These instabilities, referred to as Mode A and Mode B, occur successively with increasing Reynolds number and may be identified by a discontinuous jump in the Strouhal number as the Reynolds number is increased [Williamson 1988]. The Mode A instability may be identified by a spanwise periodicity of 3-4 cylinder diameters of the Karman vortex-street and the generation of streamwise vortex structures, formed between successive primary cores. Williamson (1996) observed that the Mode A instability existed within the Reynolds number range Re = 180 – 250; he found the critical Reynolds number of inception varies between 180-194 due to the hysteretic nature of the mode. In a Reynolds number range 230 to 250 Williamson observed that a new instability (Mode B) replaces Mode A as the dominant spanwise instability. The Mode B instability has a spanwise periodicity of around 1 cylinder diameter wavelength, which does not appear to vary appreciably with increasing Reynolds number. Williamson's (1988, 1996) observations have been verified numerically by a number of authors. Thompson et al. (1994, 1996) conducted (DNS) computational studies of the flow around a circular cylinder using a three-dimensional flow field. Their work verified the existence of both Mode A and Mode B. Barkley and Henderson (1996) conducted a Floquet analysis of the flow around a circular cylinder, verifying both the critical Reynolds number of inception of the spanwise wavelength of Mode A, and the spanwise wavelength of Mode B. The results have also been experimentally verified by Zhang et al. (1995).

An underlying assumption has been held that the instability modes identified in the wake of a circular cylinder would be exhibited in a bluff body of more general geometry. Little literature exists to verify this assumption. Zhang et al. (1995) experimentally discovered the existence of a "Mode C" instability in the wake of a circular cylinder when a tripwire was placed adjacent to a circular cylinder in a direction transverse to the fluid flow. The Mode C instability was found to have a spanwise wavelength of 1.8 cylinder diameters, and was found to occur when the trip wire was located within 1 diameter of the cylinder. Their results indicate that the suppression of the flow field near the boundary layer results in a Mode C instability to occur in preference to Mode A and Mode B. Numerical calculations performed by Zhang et al. (1995) verified their experimental observations. While their results were corroborated by the numerical Floquet analysis described by Noack et al. (1993 and 1994), who found the three-dimensional instability had a wavelength of 1.8 – 2 cylinder diameters, it now appears that this may have been due to insufficient numerical resolution. (As mentioned previously, the better resolved studies of Barkley and Henderson (1996) give wavelengths consistent with the experimentally measured values.)

Robichaux et al. (1999) discovered the existence of a third mode of instability in their Floquet analysis of the flow field around a square cross-section cylinder. While many physical features of this instability mode corresponded to Zhang's experimental work, Robichaux et al. referred to this instability as a "Mode S" instability in favor of a Mode C instability as their work did not require the existence of a trip wire. Indeed, their analysis found that the Mode S instability may not manifest because of the prior occurrence of the Mode A instability altering the base flow. Owing to the geometry of their model, it is possible that vortex shedding from the leading edge of the square cylinder interferes with the trailing-edge boundary layer and subsequently with the wake flow field, causing the Mode S instability to exist (but not dominate the flow field). However, their parameter space was limited to a Reynolds number range of ($150 < Re < 225$), and only for a square cross section cylinder; Mode S may become the dominant three-dimensional instability at higher Reynolds numbers. Previous research has not adequately answered whether the results obtained from the circular cylinder describe all three-dimensional modes observed for two-dimensional bluff bodies of general geometry?

This numerical investigation aims to observe the instability modes in the wake of a blunt plate of varying aspect ratio in order to determine if Mode A and Mode B are the dominant instabilities for a variety of geometries.

2. NUMERICAL METHOD

The bluff body geometry under investigation consists of a flat plate of finite thickness (H) and finite chord (C). Several different plates of varying aspect ratio $AR = C/H$ have been investigated. In each investigation the plate is immersed within a uniform flow field travelling in the x direction with constant velocity U. The fluid is considered to be a homogeneous incompressible Newtonian fluid. The leading edge has a streamlined elliptical profile. This profile prevents vortex shedding from the leading edge and thus allows the behaviour of the wake transition to be studied in isolation. Parameters describing the geometry of the leading edge are kept constant between plates. The trailing edges of the plates are blunt, thus providing a predetermined location from which trailing vortices are generated, and therefore simplifying post-processing analysis. An image of the plate geometry is given below (Figure 1):

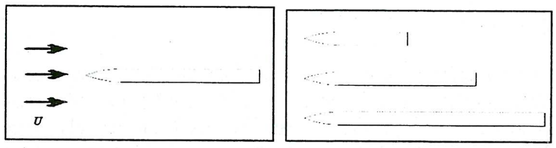

Figure 1: Left: Schematic diagram of experimental set-up; Right: Relative aspect ratios of plates.

A Floquet stability analysis is used to determine the most unstable spanwise instability mode(s) as a function of Reynolds number. Floquet stability analysis effectively investigates the stability of a two-dimensional periodic *base flow* to three-dimensional spanwise disturbances by determining from the linearised Navier-Stokes equations whether an assumed sinusoidal spanwise disturbance will grow from one base flow period to the next.

Owing to the requirement of a known two dimensional periodic base flow field, the modeling is performed in two stages; initially a two-dimensional Galerkin spectral-element code is used to determine a periodic base flow field solution (in the form of a Karman vortex street). In the second stage the (linearized) spectral-element method is used to determine instabilities in the third (spanwise) dimension. Details of the Galerkin spectral-element code may be found in Thompson et al. (1996). A detailed description of the Floquet analysis methodology may be found in Ioos and Joseph (1990).

For each plate, the parameter space consists of two variables. These are the Reynolds number based on the plate thickness (Re_{Th}), and the spanwise wavelength applicable to the stability analysis. The Floquet analysis is applied using the following approach. Starting from random perturbation fields for the perturbation velocity components and the perturbation pressure, the linearised Navier-Stokes equations are integrated forward in time. At the end of each period, the perturbation fields are normalised. Using Floquet theory it can be shown that a perturbation field can be expanded in terms of *Floquet modes*, where for any chosen spanwise wavelength, the different modes have different growth/decay rates. Thus by normalizing the perturbation fields at the end of each period, after a long time, effectively only the Floquet mode with the largest growth rate will be left. (All others grow less, or decay more, over a period than this one.) This growth rate can then be determined by calculating the amplitude of the mode at the end of a period relative to the initial amplitude. This is called the Floquet multiplier, *Fl,* and it plays an important roll in determining the stability of the two-dimensional base flow. The Floquet multiplier describes the stability of the two-dimensional flow against the selected spanwise wavelength at the given Reynolds number for the plate aspect ratio of interest. If $|\mu|$ is less than unity, the spanwise wavelength under investigation is stable at the given Reynolds number, and will not be observed experimentally. If $|\mu|$ is greater than unity, the spanwise wavelength under investigation is unstable at the given Reynolds number and should be observed experimentally (since it will grow from background noise). If $|\mu|$ is equal to unity then the Reynolds number under investigation is said to be the critical Reynolds number of inception (Re_{Cr}). In experiments, this is the lowest Reynolds number above which the instability will be observed.

3. RESULTS AND DISCUSSION

Displayed below (Figure. 2) is a diagram depicting the Floquet multiplier as a function of the Reynolds number and spanwise wavelength for a blunt flat plate with $C/H = 7.5$. Inspection of Figure 2 points to three distinct modes of instability. (The three different local maxima correspond to different instability modes.) Two of these modes correspond to Mode A and Mode B found for the flow field around a circular cylinder. However, a third mode, "Mode S" exists with a spanwise wavelength of around $\lambda = 2Th$. Similarly to the results obtained by Robichaux et al. (1999), the third mode, Mode S, does not become critically unstable. That is, the Floquet analysis predicts that Mode S should not be observed under experimental conditions. The characteristics of Mode S found in the present investigation compare favorably with those described in Robichaux et al (1999); it is therefore believed that the same instability mode occurs for both geometries.

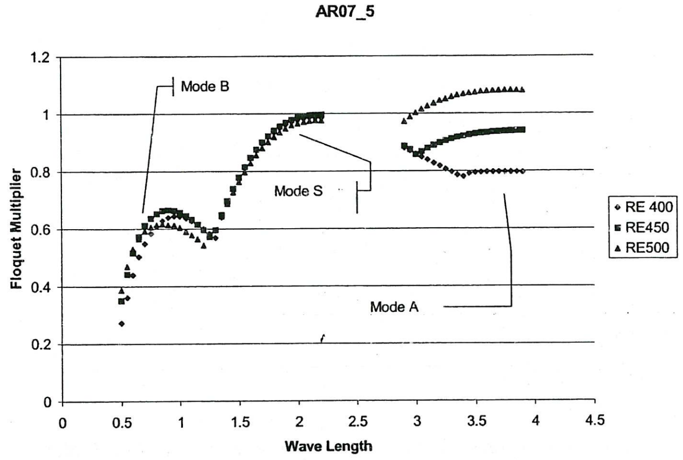

Figure 2: Floquet multiplier versus spanwise wavelength for plate $AR = 7.5$

Figure 3, displays the corresponding parameter space for a plate with $AR = 12.5$.

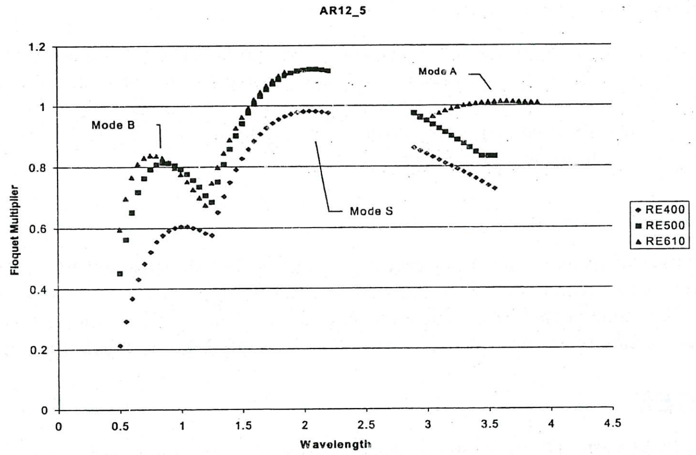

Figure 3: Floquet multiplier versus spanwise wavelength for plate $AR = 12.5$

In comparison with Figure 2, the most striking feature of Figure 3, is that an alteration in the aspect ratio of the plate has induced Mode S to become critically unstable (i.e., $|\mu| > 1$) for some regions within the parameter space (Re, λ) of interest. Furthermore, Mode S has become the most unstable wavelength. Therefore, experimentally, if the Reynolds number was increased gradually from a value below Re_{Cr}, the first spanwise instability to be noticed would be Mode S, not Mode A.

Finally, an increase in AR from 12.5 to 17.5, produces the plot shown in Figure 4 below. Once again, Mode S is the most unstable three-dimensional instability mode found. That is, it is the first to appear as the Reynolds number is increased.

When compared to the previous plate ($AR = 12.5$), the value of Re_{Cr} appears to be higher, indicating that an increased aspect ratio increases the value of Re_{Cr} for the onset of a three-dimensional wake. For all

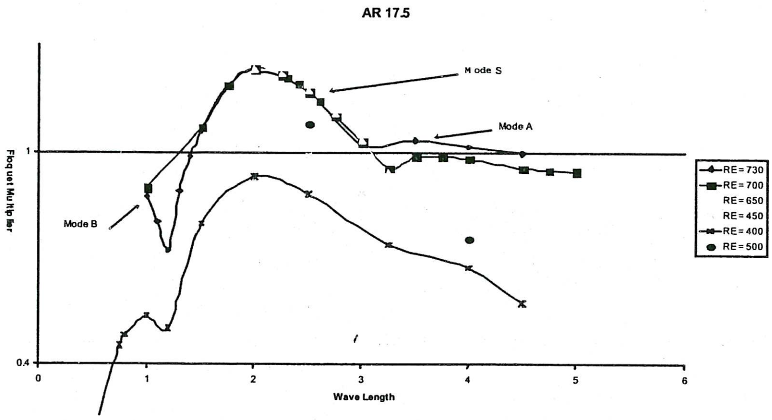

Figure 4: Floquet multiplier versus spanwise wavelength for plate AR = 17.5

aspect ratios investigated, Mode S appears to saturate towards a critical Floquet multiplier as the Reynolds number is increased. For example, for AR =17.5, a set *envelope* function describes the variation of the Floquet multiplier across the wavelength band $1.5Th < \lambda < 3Th$ for Re in the range [650, 730].

It is conjectured that the (closer to) equilibrium boundary layer profile obtained for longer bodies seems to favour the initial amplification of the Mode S instability over Mode A and B. The exact cause of this difference in preference, given relatively similar boundary layer profiles, is currently under investigation. It may be concluded that while the circular cylinder may be used successfully to provide a two-dimensional wake profile for a generic bluff body, it does not necessarily provide a three-dimensional transition scenario applicable to elongated bluff bodies.

4. CONCLUSION

The results from a Floquet analysis of the flow around a nominally two-dimensional blunt flat plate have been presented. The results indicate the presence of a third mode of three-dimensional instability, Mode S. This mode is found to have a spanwise wavelength of 1.8 – 2 plate thicknesses. For sufficiently large aspect ratio, Mode S dominates the wake flow field, becoming the principal unstable mode causing three-dimensional transition in place of Mode A, which dominates for shorter bluff bodies.

5. REFERENCES

NOACK, B. and ECKELMANN, H., "A global stability analysis of the steady and periodic cylinder wake", *J. Fluid Mech.*, **270**, 297-330, 1994.

GERRARD, J.H., "The three-dimensional structure of the wake of a circular cylinder", *J. Fluid Mech.*, **25**, 143-164, 1966.

HENDERSON, R.D., "Nonlinear dynamics and pattern formation in turbulent wake transition, *J. Fluid Mech.*, **352**, 65-112, 1997.

NOACK, B., KONIG, M., and ECKELMANN, H., "Three dimensional stability analysis of the periodic flow around a circular cylinder", *Phys. Fluids A.*, **5**, 1279-1281, 1993.

WILLIAMSON, C.H.K., "Vortex dynamics in the cylinder wake", *Annu. Rev. Fluid Mech* , **28**, 477-539, 1996.

ROSHKO, A., "On the wake and drag of bluff bodies", *J. Aeronaut. Sci.*, **22**, 124- , 1955.

BARKLEY, D. and HENDERSON, R.D., "Three-dimensional Floquet stability analysis of the wake of a circular cylinder", *J. Fluid Mech.*, **322**, 215-241, 1996.

WILLIAMSON, C.H.K., "The existence of two stages in the transition to three-dimensionality of a cylinder wake", *Phys. Fluids.*,**31**, 3165- , 1988.

ZHANG, H., FEY, U. and NOACK, B.R., "On the transition of the cylinder wake", *Phys. Fluids.*, **7**, 779-794, 1995.

ROBICHAUX, J., BALACHANDAR, S. and VANKA, S.P., "Three-dimensional Floquet instability of the wake of a square cylinder", *Phys. Fluids.*,**11**, 560-578, 1999.

THOMPSON, M., HOURIGAN, K. and SHERIDAN. J., "Three-dimensional instabilities in the wake of a circular cylinder", *Exp. Therm. Fluid Sci.*, **12**, 190 - , 1996.

IOOS, G. and JOSEPH, D.D., "Elementary Stability and Bifurcation Theory", 2nd Edition, *Springer Verlag, Berlin*, 1990.