# Experimental study of instantaneous measurement of velocity and surface topography in a wake of the circular cylinder at low Reynolds number

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# ABSTRACT

This paper studies an established technique that is capable in providing instantaneous measurements on both free surface topography and the velocity vector field of free surface flows. Testing was performed on the topography measurement by imaging static plastic wave samples over a wide range of amplitudes. The full technique was then investigated by imaging the vortex shedding generated in the wake behind a circular cylinder. The free-surface topography and the associated velocity field was reconstructed, showing a strong relationship between the two quantities.

# 1. BACKGROUND

Water wave visualisation is important in many industrial applications such as shipping and off shore mining. The generation, evolution of the water waves can cause vibration in and erosion of devices used in these industries. Therefore, the behaviour of water waves is one of the important factors that cannot be ignored in the design of such components.

Numerous fundamental studies have ben conducted to investigate free surface flows, these include the study of shallow flow around a circular cylinder by Fu and Rockwell(2005) [1], and the study of supersonic jet screech by Buchanan et al.(2007) [2], which uses the hydraulic analogy to relate the waves on a free surface to the supersonic pressure field.

According to the hydrodynamic theory, surface height relates to the pressure field in the flow. The pressure variation, which is caused by the combination of surface tension, drag force and other interactions in the fluid acceleration, results in the final topography. The instantaneous visualization of both the pressure field and the velocity field benefit the understanding of how they are related in different circumstances. It is clear that any number of fluid mechanics studies would benefit from the simultaneous measurement of both velocity field data (and related quantities such as vorticity)and the surface topography.

# 2. INTRODUCTION

A number of methods have been developed for water wave visualisation. The simplest way to reconstruct a water surface is to observe a number of floats in a grid. At some point in time, a photograph is taken and the heights of each float can be manually recorded. Similar approaches that detect water height at some point include pressure, capacitance and resistance gauges [3]. These methods interfere with the flow and hence the waves, limiting the application of these methods. Non-intrusive wave visualisation on a water table has primarily been limited to optical techniques including stereo photography, shadowgraphy, laser slope gauges and optical displacement sensors [4]. These

methods have a number of shortcomings, including poor spatial resolution, low sensitivity and yield a lack of quantitative data. Refraction techniques have been reported to have the highest sensitivity to small waves [3]. Laser slope gauges measure the gradient of a water surface by observing the refractional dislocation of a collimated laser beam between a reference (flat water) and test (wavy) condition. As the wave angle and height increases, the laser beam is deflected or dislocated further. Appropriate inclusions of lenses into the system can remove the effect of water height on dislocation and yield accurate slope information [5]. Laser slope gauges are simple to create but only give 1D spatial resolution (or 2D in time) and their application is consequently limited.

This work describes the continued development of techniques to measure the surface height of a liquid free surface. The method utilizes refraction of light at the free surface as a function of the local angle of that surface. This method utilizes Particle Image Velocimetry (PIV) interrogation of a target image viewed through the free surface. Similar methods have been developed by at least two groups. [6] have developed a system that utilizes different sources of coloured light illuminating the free surface from different angles. The result is that the reflected light is colour coded by surface angle. [7] developed a system that measures the distortion of a collimated speckle pattern. This in turn was based on the laser speckle techniques from which PIV itself has developed. This is a technically superior system, but does require the use of a good quality laser and a high degree of skill to create a collimated speckle pattern. The reliability and quantity of data such a system can supply is strongly coupled to the skill of the operator. The system outlined in this paper is based on the system in Fouras (2008) [8]. Itself based on the system described in Fouras 2006 [9].

Fouras et al(2006) [9] used a frosted glass plate to create a reference image. The method using optical distortion of frosted glass image to achieve the height estimation of the water surface. A similar technique is used by Moisy et al.(2009) [10]. By further combining this wave visualisation technique with the PIV measurement, Fouras et al. (2008) [8] advanced this technique, allowing for simultaneous measurement of velocity and surface topography. This is achieved by replacing the frosted glass with a laser sheet and a camera below the water surface.

# 3. Instantaneous Measurement of Velocity and Surface Topography

The key of this technique is to use a twin-camera imaging system. The twin cameras are individually installed at the curvy surface side (top side) and the flat surface side (bottom side) of the fluid. The twins are aligned in symmetry to provide approximately the same image field and the same magnification on the same image plane. The image plane is illuminated by a laser sheet.



**Figure 1**: Image of particle is not distorted from the real position by the flat surface. The surface gradient  $\partial H/\partial x$  refracts the light from the particle, causing a distortion of image from the real position by  $\delta x$ 

When the illuminated particles flow along the current, the bottom camera capture the non-refracted light rays scattered by the particles. Using PIV analysis, images from the bottom camera yield velocity fields of the fluid.

The top camera performs simultaneous imaging with the bottom camera. These images capture the refracted light rays scattered by the particles. Refraction occurs when the light rays pass through the wavy fluid-air interface. Using the same PIV analysis, the two simultaneous frames from the top and the bottom camera are mapped to generate a displacement vectors field. This field shows the optical distortion due to the surface gradient of the fluid.

# 3.1 Refraction of light and the height estimation algorithm

Light scattered by the particles are diffracted by the surface gradient when passing through a wavy surface. The diffracted light path causes a shift of image away from the actual particle position  $\delta x$  and  $\delta y$  in x- and y-direction respectively. Giving a reference height  $H_{ref}$  of the wavy surface, one can generate the gradient of the curved surface  $\partial H_{ref}/\partial x$  and  $\partial H_{ref}/\partial y$ , integrating the gradients gives estimated height  $H_x$  and  $H_y$ . An adjusted height  $H_{adj}$  is shifted between  $H_x$  and  $H_y$  to minimize the error between the corresponding distortion and the real optical distortion, the desired height  $H_{est}$  is selected when both errors of  $\delta x$  and  $\delta y$  are at minimum. The new estimated value  $H_{est}$  is used as the new reference height  $H_{ref}$  and the entire step is repeated until the value of  $H_{est}$  converges. A full description of this algorithm can be found in Fouras 2008 [8].

#### 4. Validation with static samples

Prior the instantaneous measurement of velocity and surface topography, the performance of the height estimation algorithm described in the previous section was tested by reconstructing surfaces of static wavy samples. An image of a frosted glass was initially captured by a CCD camera through a flat plastic sample, this undistorted glass image is used as a reference image for PIV analysis. The flat plastic was then replaced by three wavy samples with different peak to trough displacement.

Each image of the wavy samples were then compared with the reference image by proceeding ray tracing to generate an optical distortion vector field by PIV software. Using the height estimation algorithm, the distortion vector fields were integrated to give the height scalar field at each data point in the image, the free surfaces were then reconstructed from the height field using 3D plotting software. Figure 3a to 3c show the results of the reconstructed wavy surface of the samples.



**Figure 2**: Left: Frosted glass is imaged through a flat plastic sample to obtain the undistorted image; Right: The wavy plastic sample is imaged, the distorted glass image is paired with the undistorted image to yield a displacement vector field.



**Figure 3**: Surface topography of plastic samples with peaktrough distances of (top to bottom) 2.25mm, 1.15mm and 0.45mm respectively.



Figure 4: Schematic diagram of the experiment. The screen-honeycomb-screen system is used to reduce turbulence from upstream.[8]



**Figure 5**: Instantaneous surface topography in a wake of the circular cylinder at Reynolds number 98 with the cylinder just off the left of the image, with flow from left to right. The surface rendering is scaled in Z-direction (100x) to make the microscopic surface undulations observable. The vector field below shows the associated velocity field of the water at the height 45mm below the surface.

The peak to trough distance of the three samples used in the experiment was measured to be 2.25mm, 1.15mm and 0.45mm. The simulation results showed that the corresponding height of the samples are 2.27mm $\pm$ 0.04mm, 1.17mm $\pm$ 0.02mm and 0.46mm $\pm$ 0.01mm respectively, the uncertainties are about 2% and all the three results agreed well with the measured distances. The shape and the spatial distribution of the results also match with the real plastic samples. The technique was further tested by measuring the height and velocity of a free surface flow on a water table.

# 5. Instantaneous measurement of height and velocity in the wake of a cylinder

A schematic diagram of the experimental setup is shown in Figure 4. The water volume was seeded with particles and a circular cylinder of 3.9mm in diameter was inserted on splitter to reduce boundary layer effects. The wake formed downstream of the cylinder was recorded by a twin-cameras imaging system(two pco.2000 CCD cameras). The two cameras were aligned to have approximately the same field of view, the resolution of recorded images was  $1024 \times 826$  pixels, with an error in misalignment about 5 pixels. The physical size of the image was 58.5mm × 47.2mm. The particles were illuminated by a horizontal continuous laser sheet at the height of 45mm above the bed of water table.

The Reynolds number of the system was obtained by the relation Re=UD/v where U,D and v represents the upstream velocity, cylinder diameter and kinematic viscosity respectively. Experiments were performed at five different Reynolds numbers ranging between 65 and 100. 500 frames were recorded by the pairs at each Reynolds number, and time interval between each frame was 22ms.

#### 5.1 Results

Data for one Reynolds number (Re = 98), are displayed in 2D and 3D in Figures 6 and 5 respectively. The figures clearly show the accurate capturing of both the pressure field (measured as free-surface height) and velocity fields of this well known flow. Animations of the full time-series of these data highlight the accuracy and robustness of the technique in fuller detail.

#### 6. CONCLUSIONS

A detailed validation of a technique to simultaneously measure both surface topography and fluid velocity has been conducted. The results show the technique to be accurate to levels of approximately 2% without the use of averaging. Additionally, the flow in the wake of a circular cylinder has been measured at five Reynolds numbers, varying from 65 to 100. These results further validate this technique and highlight the possibility of inferring the pressure field from the free-surface topography.

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**Figure 6**: Two dimensional plot of same data as shown in Figure 5. The cylinder is located to the left of the frame. Gray scale contour shows the surface height, arrows indicate the instantaneous velocity field.

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