Non-axisymmetric spheroids: Wake transition and loss of planar symmetry

G. J. Sheard [1] , M. C. Thompson and K. Hourigan

Fluids Laboratory for Aeronautical and Industrial Research (FLAIR), Department of Mechanical Engineering, Monash University, Clayton, Victoria 3800, AUSTRALIA, Telephone: +61 3 9905 1182, Facsimile: +61 3 9905 3807

Received 15 February 2005

Abstract

Subsequent to the transitions to steady and unsteady non-axisymmetric flow in the wake of a sphere, the wake experiences a breakdown of planar symmetry. The transition process responsible for this breakdown of planar symmetry is investigated here by employing the flow past a spheroidal body with a small azimuthal asymmetry. This imposed asymmetry orients the steady double-threaded wake with the major axis of the body, whereas the unsteady wake prefers to align with the minor axis.

Key words: Sphere, Wake transitions, Planar symmetry, Non-axisymmetric flow

1 Introduction

The low-Reynolds-number transitions in the flow past a sphere are of considerable practical interest in engineering; with relevance to sedimentary, biological, and micro-scale particle-laden flows. If the Reynolds number is defined as *Re* $= Ud/\nu$, where U is the freestream velocity, d is the sphere diameter, and ν is the kinematic viscosity of the fluid, then a regular (steady-steady) transition from an axisymmetric to a non-axisymmetric wake occurs at *Re* $= 211$ [1, 2, 3, 4], and a subsequent transition to a periodic wake occurs at *Re* $= 272$ [5, 4]. Landau analysis of the non-linear evolution of these transitions has shown that the transitions occur through supercritical bifurcations [5, 4], which means that there is no hysteresis in the vicinity of the transitions. A feature of the wakes which evolve from each of these transitions is a planar symmetry [2, 4], which is observed in experiment to be sustained up to at least *Re* $= 380$ [6], and possibly up to *Re* ≈ 420 [7]. Numerical computations [8, 9] suggest that planar symmetry breaks down in the range $350 <$ *Re* < 375.

2 Methodology

In this study the breakdown of planar symmetry is investigated in the wake of a nearly-spherical body; namely a straight circular cylinder of length $0.04d$ with hemispherical ends. The flow is computed using a spectral-element/ Fourier method (used to accurately compute the flow past a sphere [4] and rings [10, 11]), and is aligned normal to the axis of symmetry of the body. Thus the body has a slight asymmetry, in contrast to a sphere, which provides a constraint on the orientation of the transition

[1] Greg.Sheard@eng.monash.edu.au

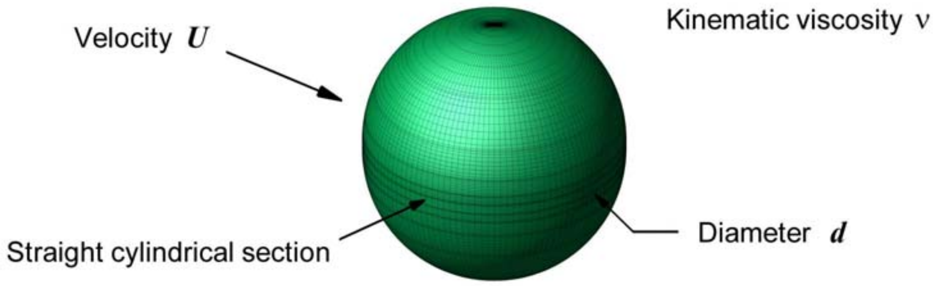

Fig. 1. The body used in the present study. The straight cylindrical section between the hemispherical ends has a length of $0.04d$, which gives it a volume and surface area 6% and 4% greater than a sphere with the same diameter.

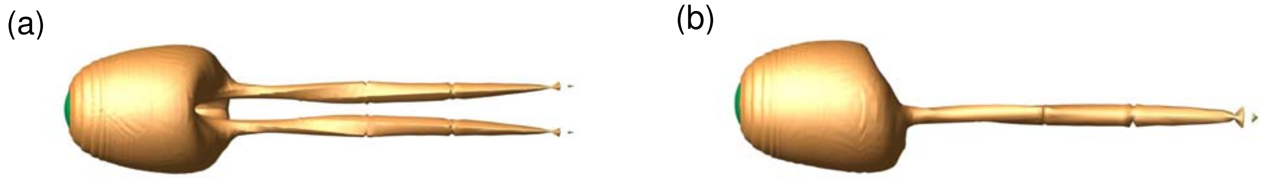

Fig. 2. The steady wake computed at *Re* $= 250$. Orthogonal top (a) and side (b) views are shown.

modes. This property is utilised in the present study to enable a closer investigation of the transition which causes the breakdown of planar symmetry. Figure 1 shows the cylinder used in this study. As verification that the body modification does not significantly alter the wakes and bifurcation scenario from those observed behind a sphere, the flow will be computed at a number of Reynolds numbers for comparison between the two bodies. The following two sections report on computations at *Re* $= 250$ and *Re* $= 300$, which are used to capture the steady double-thread wake, and the unsteady planar-symmetric hairpin wake, respectively. The findings at higher Reynolds numbers are reported in a subsequent section.

3 The Steady Wake at *Re* = 250

An isosurface plot of the final wake state reached after the evolution of the regular asymmetric transition mode is shown in figure 2. The orthogonal views verify the presence planar symmetry and illustrate the close similarity to the wake computed behind a sphere [3, 4].

It is important to observe that the wake is aligned so that the symmetry plane bisects the major axis of the body; the recirculation bubble shifts in the direction of the longer body dimension from the centre of the wake, and the streamwise vortical tails extend downstream from one of the hemispherical ends. The drag coefficient was calculated to be 0.693; within 0.53% of the coefficient obtained for a true sphere.

4 The Unsteady Wake at *Re* = 300

The computation at *Re* $= 300$ used the asymptotic wake at *Re* $= 250$ as an initial condition. An unsteady wake quickly evolved, with an orientation consistent with the preceding double-threaded wake. This observation was instable, and a slow rotation of the wake was observed. The wake reached a

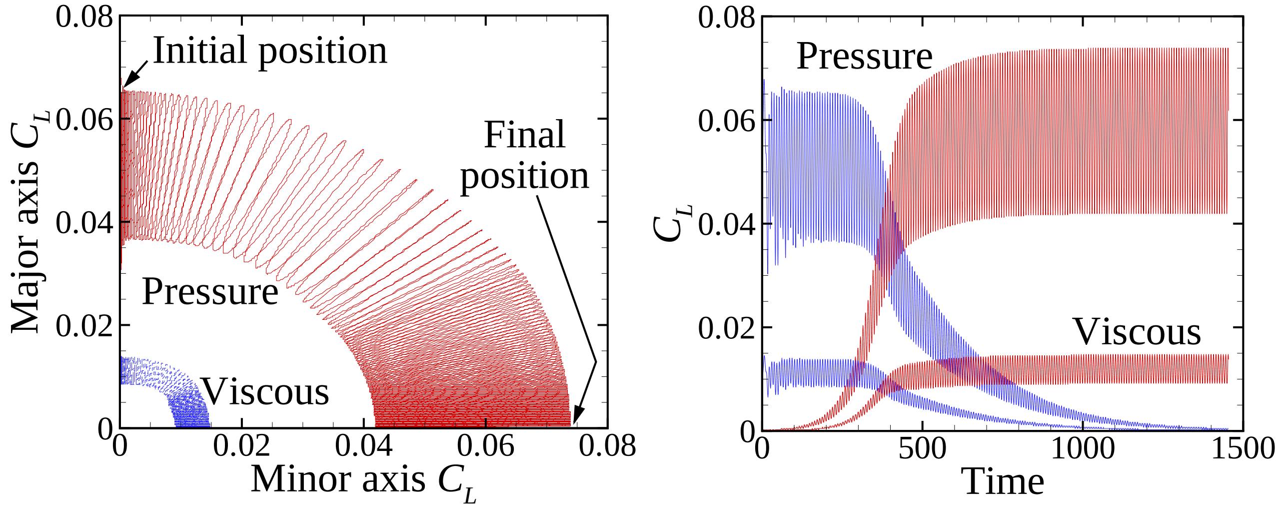

Fig. 3. Left: Phase plot of the viscous (dotted lines) and pressure (solid lines) lift coefficients acting on the spheroidal body at *Re* $= 300$. Initial and asymptotic orientations are labelled. Right: Time history of the lift coefficients measured in the major (blue) and minor (red) axis directions.

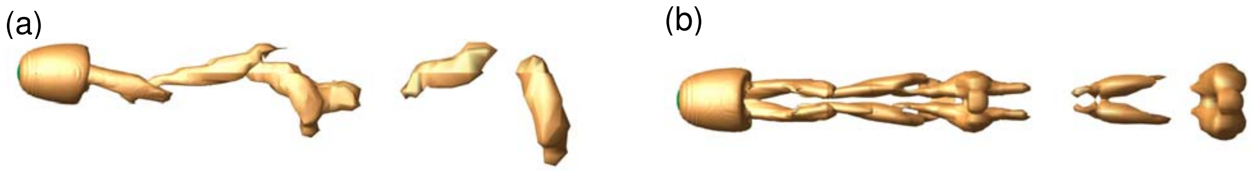

Fig. 4. Isosurface plots showing top (a) and side (b) views of the unsteady planar-symmetric wake computed at *Re* $= 300$, after rotation to its preferred orientation.

steady state after rotating through $90°$, so that the side forces imparted by the hairpin shedding was aligned with the minor axis of the body. The phase and time history plots in figure 3 show the computed change in side forces as the wake orientation changes.

The axisymmetry of a perfect sphere means that there is no preference to the orientation of the wakes produced at the Reynolds numbers employed here. Interestingly, the addition here of an asymmetry to the body shows that there are distinct preferences of orientation for the steady double-threaded wake and the hairpin wake.

The isosurface plots in figure 4 show that the wake maintains the familiar hairpin shedding pattern observed behind a sphere [8, 3, 11], and that planar symmetry is maintained at *Re* $= 300$.

5 Breakdown of Planar Symmetry

The flow was computed at a number of Reynolds numbers between *Re* $= 300$ and 370, and it is in this range that a breakdown of both the wake symmetry and periodicity occurs. These results will be explored in detail during the conference presentation, but to summarise, it was found that the loss of planar symmetry occurred in conjunction with the loss of periodicity, and no evidence of a linear evolution of the asymmetry was observed. At *Re* $= 350$ evidence of side forces normal to the plane of wake symmetry were detected, with a magnitude of approximately 10% of that of the in-plane side forces due to vortex shedding. By *Re* $= 360$, the phase-plane correlation in side forces was completely lost, suggesting that the wake was no longer sensitive to the symmetry properties imposed by the geometry.

6 Conclusions

The flow past a short cylinder with free hemispherical ends adopts the same transition process from symmetrical steady flow to unsteady three-dimensional flow observed for a sphere. The non-axisymmetric nature of the modified geometry causes a change in orientation of the non-axisymmetric wakes which evolve from the transition modes. The first-occurring regular transition adopts a plane of symmetry bisecting the body through the major axis, whereas the bifurcation to unsteady flow prefers to adopt a plane of symmetry bisecting the minor axis. Evidence of a breakdown of planar symmetry occurring in the range $350 < \mathit{Re} < 360$ was observed, and details of this transition will be provided at the conference.

7 Acknowledgments

This work was supported by the Australian Partnership for Advanced Computing (APAC) through the Merit Allocation Scheme. G.J.S. receives funding as an Australian Postdoctoral Fellow as part of an ARC Discovery Grant.

References

[1] R. Natarajan, A. Acrivos, The instability of the steady flow past spheres and disks, J. Fluid Mech. 254 (1993) 323–344.
[2] T. A. Johnson, V. C. Patel, Flow past a sphere up to a Reynolds number of 300, J. Fluid Mech. 378 (1999) 19–70.
[3] A. G. Tomboulides, S. A. Orszag, Numerical investigation of transitional and weak turbulent flow past a sphere, J. Fluid Mech. 416 (2000) 45–73.
[4] M. C. Thompson, T. Leweke, M. Provansal, Kinematics and dynamics of sphere wake transition, J. Fluids Struct. 15 (2001) 575–585.
[5] B. Ghidersa, J. Dušek, Breaking of axisymmetry and onset of unsteadiness in the wake of a sphere, J. Fluid Mech. 423 (2000) 33–69.
[6] R. H. Magarvey, R. L. Bishop, Wakes in liquid-liquid systems, Phys. Fluids 4 (7) (1961) 800–805.
[7] H. Sakamoto, H. Haniu, A study of vortex shedding from spheres in a uniform flow, J. Fluids Engng. 112.
[8] R. Mittal, Planar symmetry in the unsteady wake of a sphere, AIAA J. 37 (3) (1999) 388–390.
[9] R. Mittal, J. J. Wilson, F. M. Najjar, Symmetry properties of the transitional sphere wake, AIAA J. 40 (3) (1999) 579–582.
[10] G. J. Sheard, M. C. Thompson, K. Hourigan, From spheres to circular cylinders: The stability and flow structures of bluff ring wakes, J. Fluid Mech. 492 (2003) 147–180.
[11] G. J. Sheard, M. C. Thompson, K. Hourigan, From spheres to circular cylinders: Non-axisymmetric transitions in the flow past rings, J. Fluid Mech. 506 (2004) 45–78.