The beginning of branching behaviour during vortex-induced vibration at 2-D Reynolds numbers.

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1 Abstract

Two-dimensional simulations of flow past an elastically-mounted cylinder constrained to oscillate in the transverse direction were performed. The cylinder response during these simulations was then compared to the results from higher Re experiments. While there is a significant difference on the surface, an argument is presented that the basis of the higher-Re three-dimensional flow behaviour is present in the lower-Re two-dimensional flow.

2 Introduction

There is a large body of work on elastically-mounted cylinders constrained to oscillate transverse to a free stream. This work began with [6], and was continued by the work summarised by [10] and [2]. Some major advances were made in the understanding of response regimes and vortex-shedding modes by [9] and [7]. This, and other recent work, was summarised by [12] and [11]. The latter particularly stressed that while advances have been made in knowledge of response regimes, still little is understood of the physics of bluff structures undergoing vortex-induced vibration (VIV). The majority of the work is at Reynolds numbers where the flow is three-dimensional, except for the work of [1].

Some numerical studies of VIV have been performed, mainly at Reynolds numbers sufficiently low so that two-dimensional simulations are applicable. The Reynolds number is defined as $Re = UD/\nu$, where U is the freestream velocity, D the cylinder diameter, and ν is the dynamic viscosity. The numerical simulations include the results of [5] and [4]. These showed that while synchronisation between the vortex-shedding frequency, and the cylinder oscillation frequency at a frequency close to the structural natural frequency occurred, it occurred over a narrower range of reduced velocity, and at lower amplitudes, than three-dimensional experiments. This was supported by the comparison of 2-D, 3-D and experimental results in [3], and led to the conclusion of that study being 2-D simulations were inadequate for modelling higher-Re VIV.

It is conjectured here that while 2-D simulations of higher-Re flow ($Re > 300$) cannot faithfully model the flow at that Reynolds number, information can be gained from simulations at Reynolds number where the flow is 2-D that is applicable at the higher, 3-D Reynolds numbers. It is shown that the first stages of the upper and lower branch behaviour ([8]) can be observed, as indicated by frequency, total phase, and instantaneous phase behaviour. It is concluded that the instability that causes this branching behaviour is not a product of three-dimensionality, but is amplified by it.

3 Results and Discussion

3.1 Amplitude response

It is clear when comparing the amplitude response plots presented in figure 1a and 1b that the two-dimensional response is markedly different. The difference is characterised by three factors. Firstly, the maximum amplitude of response is much less in 2-D.

Secondly, the range of U^* that synchronisation and significant response occurs over is much less in 2-D than in 3-D. U^* is the reduced velocity, defined as $U^* = U/f_N D$, where f_N is the natural frequency including the inviscid added mass. Synchronisation also occurs at lower values of U^* in the 2-D flow. This could be due

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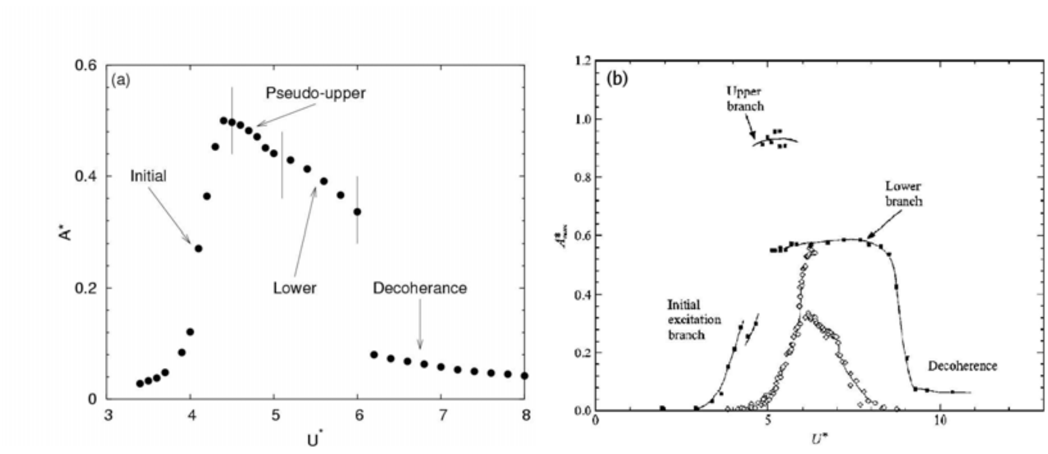

Figure 1: **(a)** Amplitude response for the 2-D simulations, $Re = 200$, $m^* = 10$, $\zeta = 0.01$. **(b)** Amplitude response for the 3-D experiments, $Re > 2000$, $m^* = 10.1$, $\zeta = 0.001$ (■). Note the higher amplitudes, longer synchronisation region, and two distinct branches for the low $m^*\zeta$ case. High $m^*\zeta$ from [6] ($\diamond$).

simply to the higher damping in the 2-D case, but similar behaviour has been observed in the comparison of [3].

Thirdly, it seems that the amplitude response in 2-D contains only one contiguous branch of significant response, whereas the response in 3-D contains two; the upper and lower branches. This seems to indicate that there is something fundamentally different between the two flows. The obvious factor is three-dimensionality, but to see such a marked branching of behaviour like this is still puzzling, if not completely surprising.

However, on closer inspection, a case can be made that prelimary signs of this split branch behaviour are present in the 2-D flow. They are not obvious for two reasons: their effect is amplified in the 3-D flow; the shorter synchronisation range sees the two branches blend together. Evidence for a pseudo-upper branch in 2-D can be found in the frequency response, and in the overall and instantaneeous phase response in the transition regions from initial to upper and upper to lower.

3.2 Frequency Response

Inspection of the frequency response plot for $m^* = 10$ (where $m^* = m/m_f$, given m is the cylinder mass and m_f is the mass of displaced fluid) shown in figure 2a clearly shows that over the synchronised range of reduced velocity, a significant shift in f/f_N occurs around $U^* = 4.8$, where f is the cylinder oscillation frequency. Before this shift, from $4.3 < U^* < 4.8$, f/f_N is slightly less than unity. After the shift, from $4.8 < U^* < 6.3$, f/f_N is greater than unity. A similar jump in f/f_N was observed by [9] (labeled as f^*) during transition from the upper to the lower branch during 3-D experiments. This is shown in figure 2b.

During the experiments, both the upper and lower branches had $f^* > 1$. However, it must be remembered that f^* is normalised by a natural frequency f_N in fluid, where the effect of the fluid is included through an added mass term, and setting the added mass equal to the inviscid added mass. It is clear that this is not the "natural" frequency of the system during VIV, as both the experimental results and the current numerical results show the system oscillates at frequencies measurably higher for the majority of synchronisation. Therefore, this distinguishing line, defined by the use of inviscid added mass, may not be a natural delineator. What is important is that at the transition from the upper to lower branch (or pseudo-upper to lower), a jump in f^* occurs in both experiment and the 2-D simulation.

3.3 Phase Response

Inspection of the overall phase response, ϕ_{ov}, plotted in figure 3a, also shows a distinctly different behaviour in the range $4.3 < U^* < 4.8$. phi_{ov} was found by finding the maximum of the cross-correlation between the lift force and displacement. This range is the only range over which the overall phase is approximately $0°$ in the synchronisation regime. This behaviour is again mirrored by the 3-D experiments of [9] reproduced in figure 3b. However, the 2-D simulations and 3-D experiments differ in the overall phase in the lower branch. The 3-D experiments see a sudden jump in phase to approximately $180°$ at the upper to lower transition. The 2-D simulations show a jump to around $40°$ at the pseudo-upper to lower transition, and then an almost linear climb

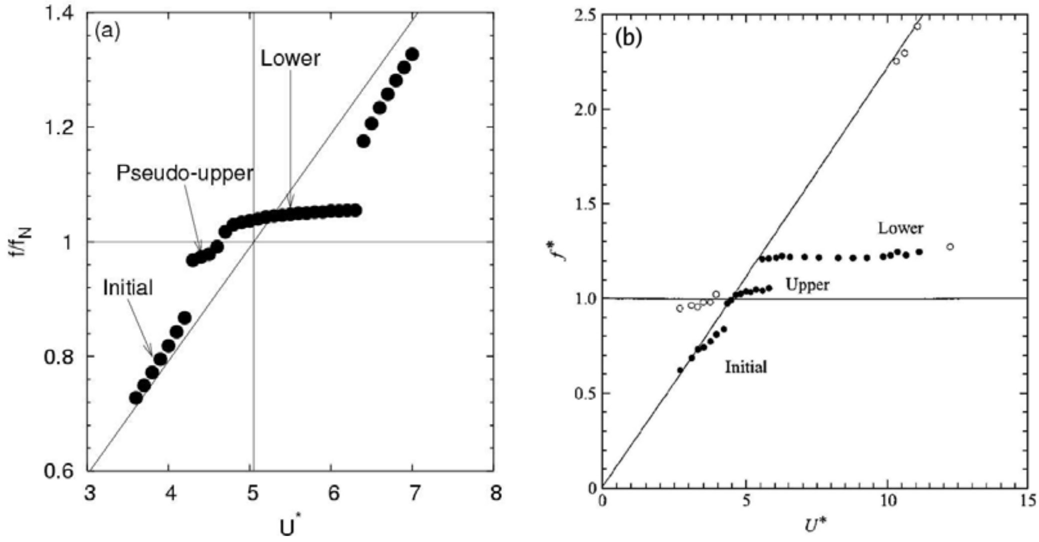

Figure 2: **(a)** Frequency response for 2-D simulations, $m^* = 10$. **(b)** Frequency response for 3-D experiments, $m^* = 3.3$, from [9]. Both show a departure from the fixed cylinder Strouhal frequency (shown by the diagonal line) to close to the natural frequency with the beginning of the upper branch, and then another jump in frequency with transition to the lower branch.

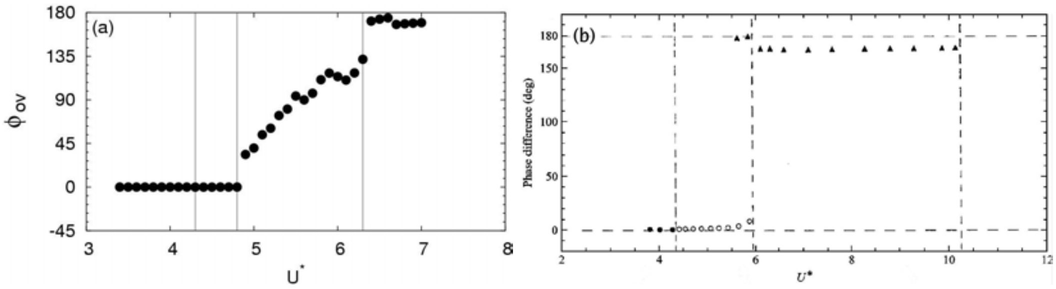

Figure 3: **(a)** Overall phase response for 2-D simulations, $m^* = 10$. **(b)** Overall phase response for 3-D experiments, $m^* = 3.3$, taken from [9]. Both show a phase of around $0°$ for the upper branch, but the transition to lower branch differs.

to around $120°$. This may simply be an effect of m^*, as simulation results obtained at $m^* = 1$ show a similar jump in phase to the 3-D experiments.

Even though the overall phase seems to characterise the pseudo-upper branch of the 2-D simulations well, its value is questionable. The use of a Hilbert transform of both lift and displacement data allows measures of instantaneous phase, ϕ, to be gained. The use of this technique shows that the phase in the pseudo-upper branch in 2-D flow is quite mobile, and is certainly not fixed with respect to time. The state that the phase seems to tend toward is dependent on oscillation amplitude, as can be seen by comparing figures 4a and 4b. The "beating" of the displacement history (which is better interpreted as a competition between two states, rather that a superposition of two frequencies) is matched by the switching of the phase. This phenomenon of non-constant amplitude oscillation is common throughout the pseudo-upper branch, and further distinguishes it from the rest of the synchronisation regime, which displays a near perfect sinusoidal oscillation.

It is interesting to note that the largest oscillations occur during this non-constant amplitude regime. It is not completely obvious whether a steady and constant amplitude was achieved during the experiments in the upper branch, but it does seem to be less stable than the lower branch response.

4 Conclusions

An indication of the upper branch, so obvious in 3-D experiments, is present in lower-Re 2-D simulations. The amplitude of response is lower, but the beginning of the instability is present. This suggests that three-

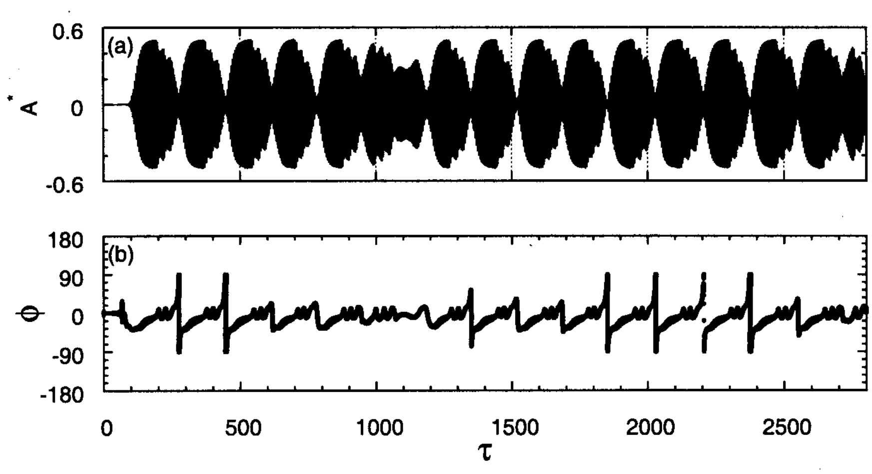

Figure 4: **(a)** Displacement history for $U^* = 4.6$. **(b)** Phase history for $U^* = 4.6$, gained through the use of a Hilbert transform. It is shown that the phase of lift to displacement is dependent on the oscillation amplitude.

dimensionality does not cause the upper branch, but amplifies it. It also means that valuable information regarding the physics of VIV can be gleaned from 2-D simulation.

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