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Three-Dimensional Floquet Stability Analysis of the Flow Produced by an Oscillating Circular Cylinder in Quiescent Fluid

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Abstract

The primary three-dimensional instabilities of the flow generated by a cylinder oscillating in quiescent fluid at low amplitudes and frequencies of oscillation are examined using Floquet stability analysis and direct numerical simulation (DNS). Experimental visualisations have observed the formation of a number of regimes which, in part, have been classified by the nature of the three-dimensional structures formed. Using a combination of Floquet analysis and DNS the transition from two-dimensional to three-dimensional flow is studied for the onset of three of these regimes. Neutral stability curves and the spanwise wavelength at the onset of these instabilites are reported for values of the dimensionless oscillation amplitude (Keulegan-Carpenter number) $KC < 10$ and the frequency (Stokes number) $\beta < 100$. Vorticity isosurfaces of the three-dimensionally unstable flows studies are presented.

Introduction

The harmonic oscillation of a cylinder in a quiescent fluid is controlled by two dimensionless control variables: The Keulegan–Carpenter $KC = 2\pi a/D$ and the Stokes number $\beta = fD^2/\nu$ where a is the amplitude of motion, D is the cylinder diameter and f is the frequency of oscillation. At very low amplitudes and frequencies of motion the flow about the cylinder is two-dimensional, symmetric about the axis of oscillation and synchronous with the oscillation period. A secondary streaming flow, which can be observed in the long term average, is also seen to form and it streams out symmetrically along the axis of oscillation. Experimental observations of the flow generated by this motion have revealed a number of unique structures formed in the cylinder wake which break these initial symmetries. Honji (1981) presented visualisations of a 'streaked flow' regime where separated dye sheets were shown to form chains periodically along the cylinder span. Tatsuno & Bearman (1990) subsequently identified a large number of regimes within the chosen parameter space through visualisation of the flow structures which permitted classification into distinct regimes. The nomenclature introduced by Tatsuno & Bearman (1990) in labelling these regimes $A^* - G$ will be utilised herein. The visual observations of Honji (1981) and Tatsuno & Bearman (1990) demonstrated that symmetry breaking transitions from the initial symmetric base flow occur through both two and three dimensional instabilities and result in the formation of a number of distinct flow regimes.

The two-dimensional symmetry breaking transition was explored in Elston, Sheridan & Blackburn (2004) using Floquet analysis. It was shown that there is a single curve of marginal stability in the (β, KC)-space that closely matches the locus of points found for the transitions between regimes $A-D$, $A-C$ and $B-E$. In the $B-E$ transition the base flow has been shown to be already three-dimensional (Honji's streaked flow) however the two-dimensional transition matched this transition remarkably well. Two distinct forms of bifurcation were observed along this neutral stability curve. The Floquet multipliers for the $A-D$ transition are real, resulting in a perturbation to the base flow that was synchronous with the cylinder's oscillation period. In the $A-C$ and $B-E$ transition cases the Floquet multipliers occur in complex-conjugate pairs. These transitions were found to occur through a Neimark–Sacker bifurcation, with the imaginary component of the multipliers relating to a new secondary period, T_s, leading to states which are quasi-periodic.

In the results presented here we use Floquet stability analysis to obtain the locus, in (KC, β) control space, of the three-dimensional symmetry breaking transitions and to determine the wavenumbers at the onset of these instabilities. DNS is used to study the case where it is not possible to use Floquet analysis to determine the onset of a three-dimensional transition.

Computational Methodology

In the present implementation the periodic flow to be tested for stability (the 'base flow') is obtained using DNS. The DNS employed a spectral element spatial discretisation to solve the two-dimensional incompressible Navier–Stokes equations in an accelerating reference frame attached to the cylinder (Blackburn & Henderson 1999). In some cases the symmetry of the base flow about the axis of oscillation was enforced by applying a set of symmetry boundary conditions along this axis. In this case at $x = 0$ the boundary conditions were: $u = 0$ and $\partial v/\partial n = 0$ combined with a high-order pressure boundary condition. Fourier series interpolation was subsequently used to reconstruct the 64 field dumps, equi-spaced in time, of the base flow.

The linear stability of the symmetrical periodic base flow to infinitesimal perturbations was then calculated using Floquet stability analysis. In Floquet stability analysis a T-periodic flow, $\mathbf{U}$, is examined in conjunction with a perturbation, $\mathbf{u}'$, to the flow to determine whether the perturbation grows or decays in time. The evolution equations for the perturbation flow are the Navier–Stokes equations linearised about the T-periodic base flow. Perturbation solutions, $\mathbf{u}'$, can be written as a sum of components $\tilde{\mathbf{u}}(t)\exp\sigma t$ where $\tilde{\mathbf{u}}(t)$ are the T-periodic Floquet modes. Equivalent to the Floquet exponents σ are the Floquet multipliers μ, where $\mu = \exp\sigma t$. In this technique the evolution of a system governed by the linearised Navier–Stokes equations is examined every time T to extract the leading eigenpairs of the evolving perturbation, $\mathbf{u}'(x,y,t)$, subject to the forcing of the periodic base flow, $\mathbf{U}(x,y,t)$, using a Krylov subspace method (Barkley & Henderson 1996). The Floquet simulations were run on the full size domain with a Gauss–Lobatto–Legendre (GLL) polynomial interpolant order of 8, the selection of which is detailed in Elston et al. (2004). Domain sizes of $40D \times 40D$ and $40D \times 80D$ were used in the simulations.

In determining the primary three-dimensional symmetry breaking characteristics of this system the underlying nature of the two-dimensional base flow must be considered. It is known that the underlying base flow had the potential to have broken the two-dimensional spatial symmetry about the axis of oscillation (Elston et al. 2004) and the impact of this will be considered by enforcing this symmetry. An additional limitation is that Floquet analysis can only be used where the base flow is periodic. As a consequence DNS was used to determine the stability characteristics for the transition $A-C$ where the present authors had previously determined the base flow to be quasi-periodic (Elston et al. 2004).

Results and Discussion

The boundaries determined for the onset of a three-dimensional instability in (KC,β)-space are summarised in figure 1(a). Three distinct forms of three-dimensional instability boundaries are shown. In the case of the $A-D$ transition (■) the base flow had already broken reflection symmetry about the oscillation axis and the resulting flow appeared to be immediately three-dimensionally unstable. In the second case where the underlying flow was naturally two-dimensionally symmetrical (the A^*-B transition, •), our results are in excellent agreement with the boundary established experimentally by Tatsuno & Bearman (1990). In the third case presented (○), the two-dimensional symmetry of the base flow was enforced enabling the calculation of this boundary despite being above the two-dimensional symmetry breaking curve. This permitted the authors to identify an additional stability curve that would have occurred had the two-dimensional flow not broken symmetry. The critical wavenumbers at the onset of the three-dimensional instabilities are shown in Figure 1(b). The values for the $A-D$ transition are in remarkable agreement with the experimental values obtained by Tatsuno & Bearman (1990). The values obtained for the A^*-B transition are in reasonable agreement also. As the third case with enforced symmetry is not physically realisable there are no comparable experimental results.

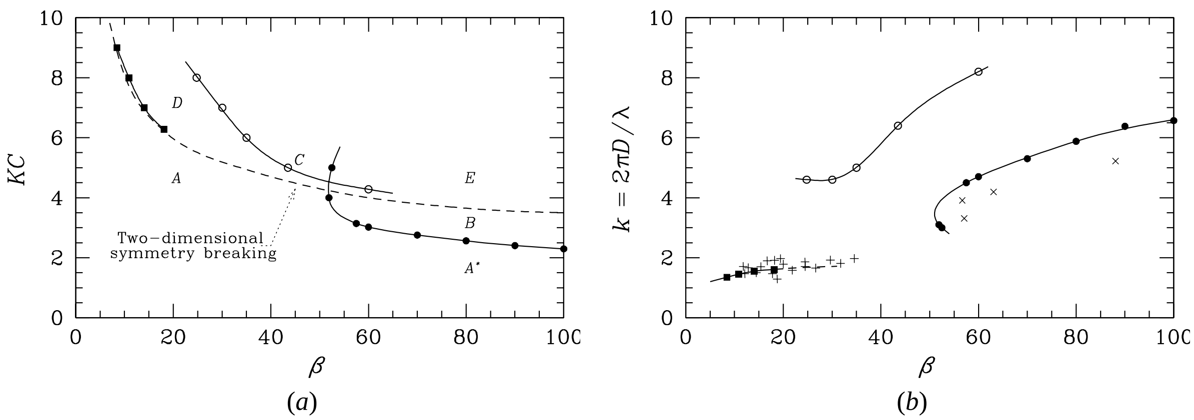
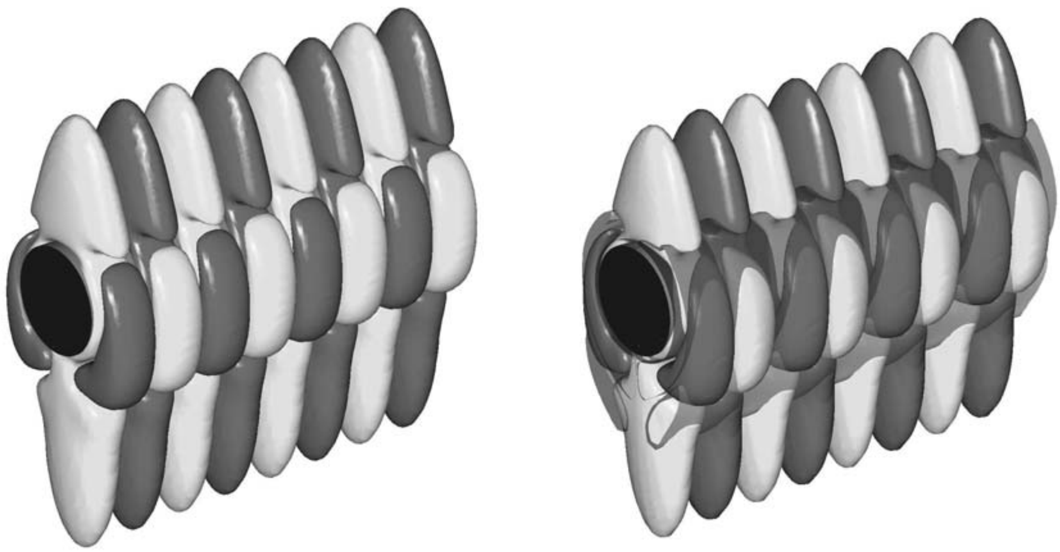


Figure 1: Marginal stability curves for three-dimensional modes. Letters A^*-E denote the approximate location of the regimes identified by Tatsuno & Bearman (1990). For data labelled •, ○, the base flow had two-dimensional reflection symmetry. For data labelled ■, the base flow had broken two-dimensional symmetry. In panel (*a*), the curve of two-dimensional symmetry breaking from (Elston et al. 2004) is shown as a dashed line. In (*b*), wavenumber data from Tatsuno & Bearman (1990) are represented by × (regime B) and + (regime D).

The instantaneous vorticity isosurfaces for the unstable Floquet mode and the corresponding DNS result for regime B are shown in figure 2. The DNS visualisation was obtained at saturation and spanwise discretisation of a single spanwise length was achieved using eight Fourier modes. Four repetitions are shown. A visual comparison between the linear and nonlinear results shows that the nonlinear interactions results in the vortex pairs rolling up towards each other as they sweep past the cylinder's shoulders, additionally the vorticity structures above and below cylinder can be seen to be now slightly inclined to the $x-y$ plane. These structures are consistent with the visualisations of Honji (1981) and Tatsuno & Bearman (1990) where 'streaked flow' was observed. The wavelengths predicted by Floquet analysis are in reasonable agreement with the results of Tatsuno & Bearman (1990).

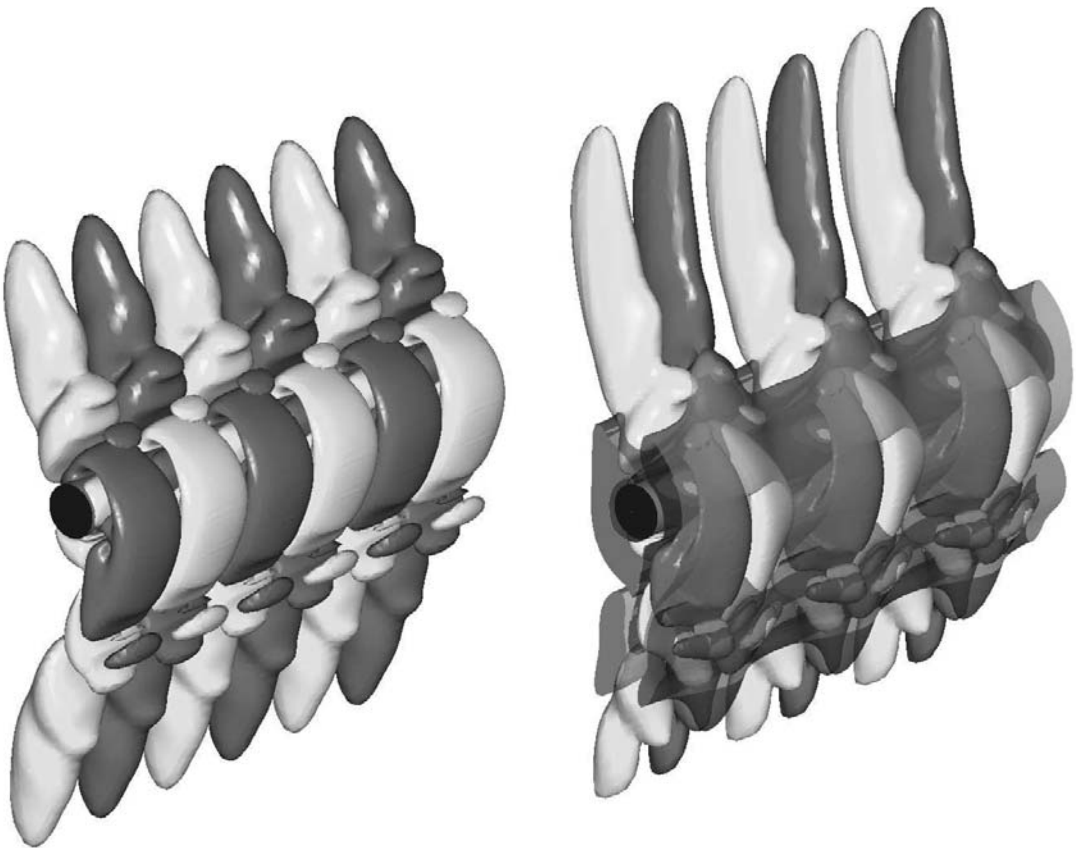
A comparison of the instantaneous vorticity contours for the Floquet mode and DNS computations in regime D, figure 3, highlights the impact of nonlinear interactions. In this case the underlying two-dimensional flow has broken symmetry (Elston et al. 2004), however the DNS flow and Floquet mode are still synchronous with the cylinder's oscillation. The nonlinear effect of pairing of oppositely signed vorticity along the span can clearly be seen in the DNS result. The



(*a*) (*b*)

Figure 2: Instantaneous vorticity isosurfaces for the regime B instability at (β=80, *KC*=2.6, $k = 5.88$), showing (*a*) the Floquet mode and (*b*) DNS result. Four spanwise repetitions are represented, at the instant when the cylinder is at $y_{\max}$. The solid isosurfaces show *y*-component vorticity of equal magnitude but opposite signs, while additionally in (*b*) translucent isosurfaces show *z*-component vorticity.

wavelengths predicted by Floquet analysis are in excellent agreement with experimental visualisations, as can be seen in figure 1(b). The three-dimensional instability that arose for the transition $A-D$ appears to be dependent on the breaking of the two-dimensional symmetry about the oscillation axis. When this was prevented, by enforcing symmetry about the oscillation axis in the base flow, the onset of a three-dimensional instability was suppressed until much higher values of *KC* or β (figure 1).



(*a*) (*b*)

Figure 3: Instantaneous vorticity isosurfaces for the regime D instability at (β=14.15, *KC*=7, $k = 1.75$), showing (*a*) the Floquet mode and (*b*) DNS result. Three spanwise repetitions are represented, at the instant when the cylinder is at $y_{\max}$. The solid isosurfaces show *y*-component vorticity of equal magnitude but opposite signs, while additionally in (*b*) translucent isosurfaces show *z*-component vorticity.

The transition from regime A to C was problematic in that it had been previously established that a two-dimensional quasi-periodic bifurcation had already occurred to the base flow and that Floquet analysis is only applicable to the study of periodic flows. As a result DNS was employed exclusively to study this problem. Isosurfaces of vorticity of a regime C flow are shown in figure 4. While this set of visualisations was obtained for a wavenumber of $k = 0.5$, it was found that the flow is unstable to a band of wavenumbers that emanate from $k = 0$. The band of unstable wavenumbers was found to increase with increasing KC. The isosurfaces presented could result in dye visualisations such as those seen in figures 15 and 16 of Tatsuno & Bearman (1990), with their characteristic chevron patterns.

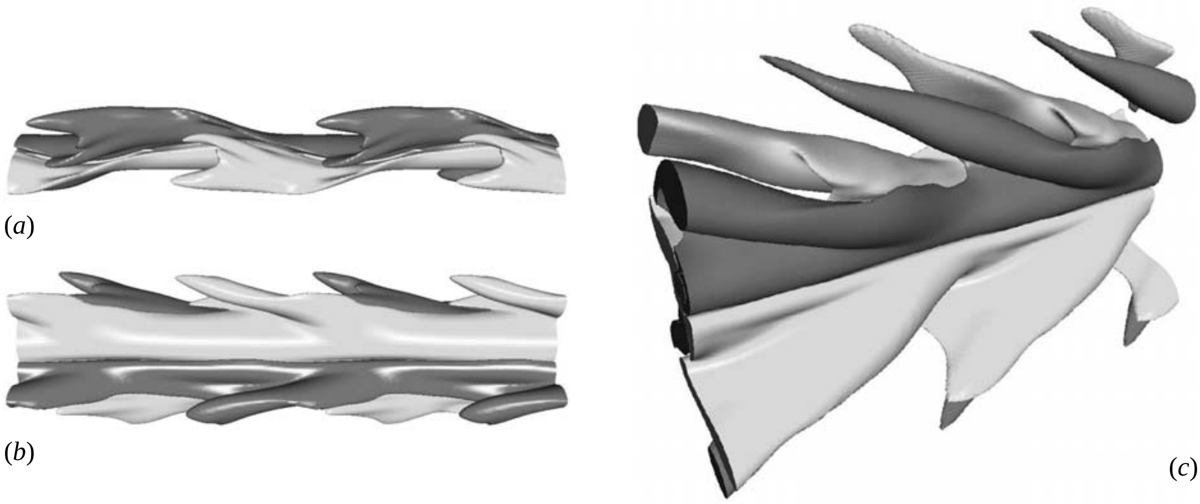


Figure 4: Instantaneous vorticity isosurfaces for the regime C instability, obtained from DNS at (β=40, KC=4.8, $k = 0.5$). Two spanwise repetitions of equal-magnitude positive and negative isosurfaces of spanwise vorticity component are shown: (*a*), top; (*b*), side and (*c*) perspective views. Isosurface magnitudes in (*c*) are lower than for (*a*, *b*).

Conclusions

The predictions of the onset of three-dimensional instabilities in (KC, β)-space found through Floquet analysis are in excellent agreement with those found in the experimental visualisations of Honji (1981) and Tatsuno & Bearman (1990). For the case where the two-dimensional reflection symmetry had broken, the critical wavenumber was in excellent agreement with experimental results. The onset of this three-dimensional instability was contingent upon the underlying two-dimensional flow having broken symmetry. When the two-dimensional symmetry about the oscillation axis was enforced the onset of a three-dimensional instability was shifted to higher values in the (KC, β) plane. When the underlying two-dimensional flow was naturally symmetrical the onset of a three-dimensional instability was found to coincide with experimental results. The critical wavenumbers predicted in this case were in reasonable agreement with experimental results. The transition from regime A to C is marked by the onset of a three-dimensional instability in experimental visualisations. However, as the underlying two-dimensional flow had become quasi-periodic Floquet analysis was not applicable. DNS of the cylinder wake for this case yielded flow structures resembling those reported in experimental visualisations.

Acknowledgements

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