Tracked flame simulation for a type Ia supernova

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Abstract

In this paper, we present a numerical model for a Type Ia supernova explosion. Our method is based on explicit tracking of the flame front, which is critically important to the accurate modeling of turbulent thermonuclear combustion.

Keywords: Front tracking; Supernova; Combustion

1. Introduction

We study turbulent combustion in Type Ia supernovae. These supernova – the brightest and most frequently observed – are believed to be thermonuclear explosions of white dwarfs. Type Ia supernovae are important sources of energy and chemical elements deposited into the interstellar medium, and they are important distance indicators to measure the expansion of the universe. The chemical composition, the density and velocity of the ejecta, and the strength and brightness of the burning are all determined by the speed of thermonuclear burning [1,2]. The problem of turbulent combustion may also be important for other astrophysical objects such as novae, X-ray bursters, collapsing white dwarfs, etc. To determine the speed of turbulent combustion from first principles is of considerable interest.

Despite more than thirty years of intensive investigation, the physics whereby the carbon-oxygen core of a star near the Chandrasekhar mass explodes as a Type Ia supernova (SN Ia) is still debated. This indeed is a hard problem because the nuclear flame propagates in an extensive medium in which gravity plays a role and several instabilities have time to develop over a large range of length scales. Our understanding of SN Ia explosions is far from complete. To predict the explosion outcome, one needs to model the propagation of thermonuclear burning inside the exploding star. The mechanism and the speed of thermonuclear burning in SN Ia remain an unsolved theoretical problem. A recent review by Niemeyer et al. [3] contains a large list of relevant publications.

In order to model turbulent thermonuclear combustion accurately, we track the flame front. The reasons for this are simple. First, it is important to reproduce the geometry of the flame front as accurately as possible, because its effective surface area also determines the rate of fuel consumption. Secondly, since nuclear reactions are very sensitive to temperature and their rates depend on a very high power of T, mixing fuel and ashes numerically in certain mesh points on both sides of the front and smearing out the temperature gradient there will lead to large errors in the (local) energy generation rates. Therefore, if in an algorithm one wants to follow the nuclear transmutations explicitly one is forced to guarantee that temperature jumps due to fast reactions are tracked with high precision. Here we propose to conduct numerical simulations for a Type Ia supernova by applying the flame tracking algorithm which will be summarized in the next section. For the equation of state (EOS), we will start with a gamma law gas as a first approximation since the real EOS is not far from $\gamma = 4/3$.

2. Tracked flame model

The liberated energy from combustion is determined from the molecular or nuclear binding energy of the unburnt gas. We denote the chemical or nuclear (binding) energy by q and introduce the total specific energy

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$$\tilde{E} = e + \frac{1}{2}\vec{v}\cdot\vec{v} + q = E + q$$

with the specific internal energy e, the pressure p, and the velocity $\vec{v}$. The relations governing the transition from unburnt to burnt gas are derived from the three conservation laws of mass, momentum, and energy in the same way as were the relations governing a shock transition.

Here we solve the following 2D axisymmetric Euler equations in cylindrical coordinates (r, z):

$$\rho_t + (\rho v_0)_r + (\rho v_1)_z = -\frac{1}{r}\rho v_0 \tag{1}$$

$$(\rho v_0)_t + (\rho v_0^2)_r + (\rho v_0 v_1)_z + p_r = -\frac{1}{r}\rho v_0^2 + \rho g_0 \tag{2}$$

$$(\rho v_1)_t + (\rho v_1 v_0)_r + (\rho v_1^2)_z + p_z = -\frac{1}{r}\rho v_1 v_0 + \rho g_1 \tag{3}$$

$$(\rho E)_t + (\rho E v_0)_r + (\rho E v_1)_z + (p v_0)_r + (p v_1)_z = -\frac{1}{r}\rho v_0 - \frac{1}{r}\rho v_0 + \rho(g_0 v_0 + g_1 v_1) \tag{4}$$

where ρ is the mass density of the fluid, $\vec{v} = v_0\vec{r} + v_1\vec{z}$ is the fluid velocity, and $\vec{g}$ is the gravity. The equations (1)–(4) describe the conservation laws of mass, momentum, and the total energy. The system is closed via a thermodynamic equation of state relating density, pressure, and energy, most commonly though a functional relation $p = p(\rho, e)$. The algorithm for tracking a combustion front is an extension of the front tracking algorithm for a non-reacting gas interface [4], to a deflagration flame front. The idea is motivated by [5,6]. For algorithms for tracking a detonation front, we refer to Bukiet et al. [7].

Point propagate [8] is a basic front-tracking operation. This operator computes the time-advanced position and state of the front. The currently implemented algorithm uses local dimensional splitting to decompose the equations of motion into components normal and tangential to the interface. Let $\vec{N} = N_0\vec{r} + N_1\vec{z}$, $\vec{T} = T_0\vec{r} + T_1\vec{z}$ be the normal and tangential unit vectors at some point on the front. Then $\vec{v}$ can be rewritten as $\vec{v} = v_N\vec{N} + v_T\vec{T}$, $v_0 = v_N N_0 + v_T T_0$ and $\vec{g} = g_N\vec{T} + g_T T$. The projection of the equations (1)–(4) onto the normal direction can be written as

$$\rho_t + \frac{\partial}{\partial N}(\rho v_N) = -\frac{1}{r}\rho v_N N_0 \tag{5}$$

$$\rho v_N)_t + \frac{\partial}{\partial N}(\rho v_N^2 + p) = -\frac{1}{r}\rho v_N^2 N_0 + \rho g_N \tag{6}$$

$$(\rho v_T)_t + \frac{\partial}{\partial N}(\rho v_N v_T) = -\frac{1}{r}\rho v_T v_N N_0 \tag{7}$$

$$(\rho E)_t + \frac{\partial}{\partial N}(\rho E v_N) + \frac{\partial}{\partial N}(p v_N) = -\frac{1}{r}\rho E v_N N_0 - \frac{1}{r}p v_N N_0 + \rho g_N v_N \tag{8}$$

The equations projected onto the tangential direction $\vec{T}$ can be written similarly. Fig. (1a) shows the basic stencil of states used to compute the contribution of the normal component of flow. The states sl_0 and sr_0 denote the left and right states at the point to be propagated. States sl_i and sr_i are interpolated at distances Δn in the direction normal to the front. The projection of these states onto the line normal to the front is used to move the interface point and compute a pair of time updated left and right states at the front.

For tracking the deflagration combustion front, we should assume the laminar flame speed s_f and the nuclear energy density $q_0(q_1)$ for unburned (burned) gas. The key element in the point propagation algorithm is

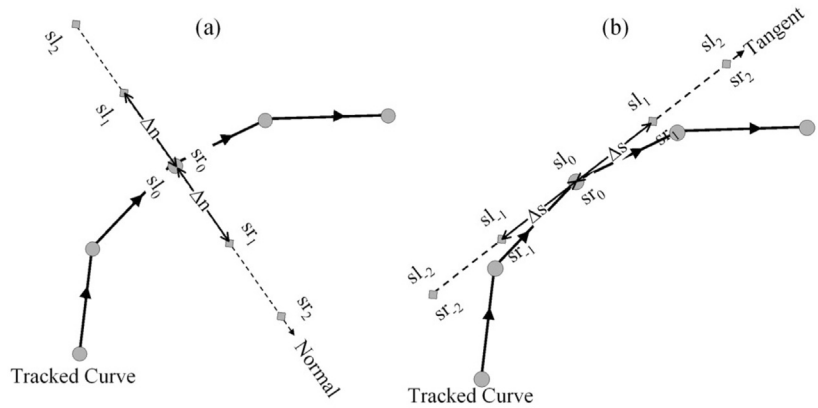

Fig. 1. A schematic showing the stencil of states used in propagating a front point. The normal propagate stencil is shown in (a), while (b) shows the stencil used in the tangential update.

the Rankine-Hugoniot (RH) jump condition. The RH relations across the front are given by

$$(\rho v_N) = s_f(\rho)_{\text{jump}} \tag{9}$$

$$(\rho v_N^2 + p)_{\text{jump}} = s_f(\rho v_N)_{\text{jump}} \tag{10}$$

$$(\rho v_N v_T)_{\text{jump}} = s_f(\rho v_T)_{\text{jump}} \tag{11}$$

$$(\rho E v_N + p v_N)_{\text{jump}} + (\rho q v_N)_{\text{jump}} = s_f(\rho E)_{\text{jump}} + s_f(\rho q)_{\text{jump}} \tag{12}$$

where $E = e + \frac{1}{2}\vec{v}\cdot\vec{v}$, $(x)_{\text{jump}} = x_r - x_l$, and x_l and x_r are the values at left and right of the front respectively.

After front propagation, we use standard finite difference methods to perform the tangential sweep and interior update. An essential question remaining to be answered is how to couple the front information to the interior states. The main idea is that we treat the tracked fronts as internal time dependent 'boundaries'. Therefore the fronts divide the domain into several subdomains. We solve the problem inside each subdomain with front states (from the correct side) as boundary conditions and use ghost cell extrapolation to compute the finite difference stencils [9]. In that way, finite differencing is never performed across the front, and the flow gradient remains perfectly sharp at the front.

3. Numerical simulation

Our simulations are carried out for the following nuclear reaction:

$$C \rightarrow N_i + \text{energy(q)}$$

The macroscopic flame propagation speed and even the microscopic combustion process in a flame are not yet well understood in many turbulent burning regimes. Fortunately the situation is not too complicated for supernova Ia, since the flame starts in the laminar regime and then quickly changes to the flamelet regime with high turbulent intensity and there exists an accepted ansatz for the burning speed for both regimes [10]. For the laminar flame speed an approximation function was derived by Timmes et al. [11] by performing fully resolved, microscopic combustion simulations in white dwarf matter for many different parameter sets. In this paper the laminar flame propagates at

$$s_l = 9.2 \times 10^6 \text{cm/s} \left(\frac{\rho}{2. \times 10^9 \,\text{g/cm}^3} \right)^{0.805} (2 \times X(C))^{0.889}$$

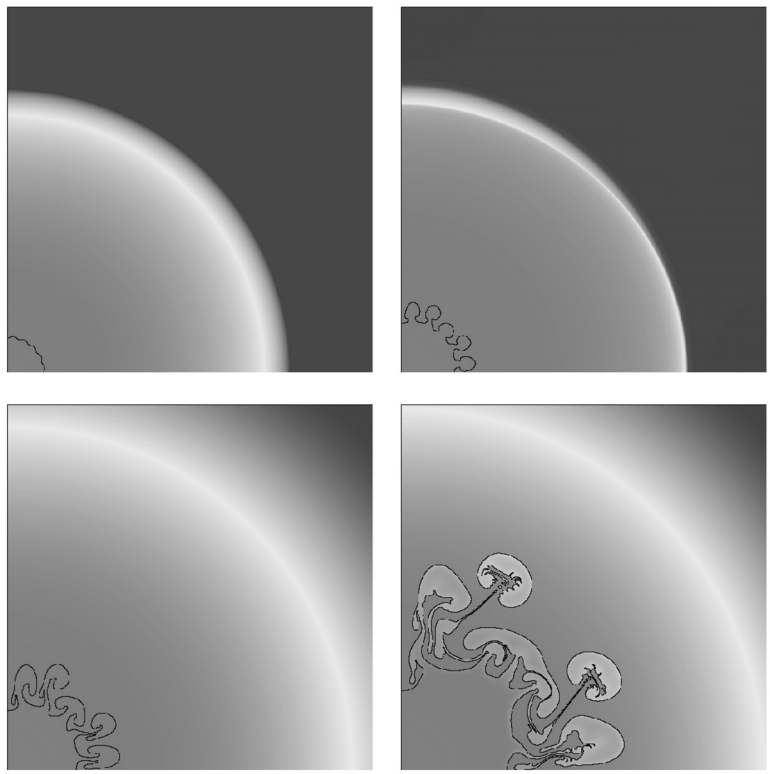

Fig. 2. Evolution of combustion front in SN Ia explosion.

where ρ is the density of unburned mass, and $X(C)$ is the carbon mass fraction. The above expression serves as a lower limit for the flame speed throughout the simulation.

Our (r, z) computational domain is $[0, r_1] \times [0, z_1]$, with $z_1 = r_1 = 3 \times 10^8$ cm. The origin is denoted by $P_0 = (0, 0)$. Let ρ denote the distance from any point in the computational domain to P_0. The flame surface is located at the perturbed circle $\rho = \rho_0 + a_0 \cos(m\phi)$ with a_0 the initial amplitude, m the frequency, and ϕ the azimuthal angle from the r-axis to the ray joining P_0 to the point at the flame front. In our experiment, the inner fluid is burned and the outer fluid is unburned. Due to the rotational symmetry about the z-axis, we are considering an axisymmetric perturbed spherical exploding problem. Reflecting boundary conditions are used at the left and lower sides, i.e. the $r = 0$ and $z = 0$ axes. Flow-through boundary conditions are applied at the upper and right boundaries of the domain so that outbound waves will exit the domain. The physical parameters for our simulations are: the initial amplitude to wavelength ratio $a_0/\lambda = 0.06$, the frequency $m = 5$, the central density is 2.9×10^9 g/cm^3 and the energy release $q = 9.28667 \times 10^{17}$ erg/g.

Fig. 2 shows the evolution of the flame front. We see that mushroom type complex interfaces have been developed in later times since the flame is unstable subject to Rayleigh-Taylor instability resulting from the buoyancy of hot, burned fluid with respect to the dense, unburned material [2,12,13]. Rayleigh-Taylor instability is the dominating effect on turbulent combustion in supernova since it increases the effective surface area of the flamelets, thereby the rate of fuel consumption.

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