

Three fast computational approximation methods in hypersonic aerothermodynamics

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Abstract

The applicability of three numerical approximation methods of solving the Navier-Stokes equations (local stagnation streamline approximation, ‘parabolized’ equations, and the thin-viscous-shock-layer approach) have been analyzed to study nonequilibrium hypersonic viscous flows near blunt bodies. These approximations allow reducing the calculation time by factor of 10 in comparison with the time needed to solve the full Navier-Stokes equations. The study demonstrates a significant influence of regularization procedures on the approximate solutions.

Keywords: Regularization algorithms; ‘Parabolized’ Navier-Stokes equations; Viscous shock layer; Nonequilibrium hypersonic flows

1. Introduction

Numerous methods [1,2] that require significant computational resources have been developed to study flowfield parameters around hypersonic vehicles. In the present study, fast computational approximation methods (local stagnation streamline approximation [3,4], ‘parabolized’ Navier-Stokes equations [4,5], and the thin-viscous-shock-layer approach [6,7]) are analyzed in the cases of hypersonic flows around a sphere and a blunt plate at angles of attack. Numerical approximate solutions are compared with the solutions of the full Navier-Stokes equations [8,9], experimental data [10,11] and the results of the direct simulation Monte-Carlo technique [12] at moderate Reynolds numbers $1000 > Re_0 > 10$. Effective regularization algorithms [3,4,5,6,7] were developed.

2. Numerical solutions of the Navier-Stokes equations

The Navier-Stokes equations [9], relaxation equation, and expressions for heat and energy diffusion fluxes [13] are used here to describe the airflow with rotational relaxation. The undisturbed upstream flow and ‘free flow’ conditions [8] at the distances far from the body

were used. The slip, temperature jump, rotational energy jump, and the diffusion velocity slip were specified on the body surface [3]. The conservative finite-difference scheme [8] and Seidel’s implicit method [9] were used in the study.

The numerical solutions for translational (T_t) and equilibrium (T_{eq}) temperatures, and nonequilibrium rotational energy (E_r) at Reynolds number $Re_0 = \rho_\infty U_\infty a / \mu(T_0) = 14.4$, Mach number $M_\infty = 6.5$, and temperature factor $t_w = 0.3$ are shown in Fig. 1 (dashed lines). The numerical results for E_r correlate well with experimental data [10].

The Stanton numbers St at the spherical stagnation point were calculated at $M_\infty = 15$, $t_w = 0.15$, and various Reynolds numbers. The solutions of the Navier-Stokes equations with slip and non-slip boundary conditions are shown in Fig. 2. The results correlate well with experimental data [11], numerical DSMC data [12] and boundary-layer solution. The slip conditions must be included at $Re_0 < 20$.

3. Local approximate solutions of the Navier-Stokes equations

A local similitude character of flow near the stagnation streamline is used for transforming the Navier-Stokes equations into the system of ordinary differential equations [4]. This simplification drastically decreases

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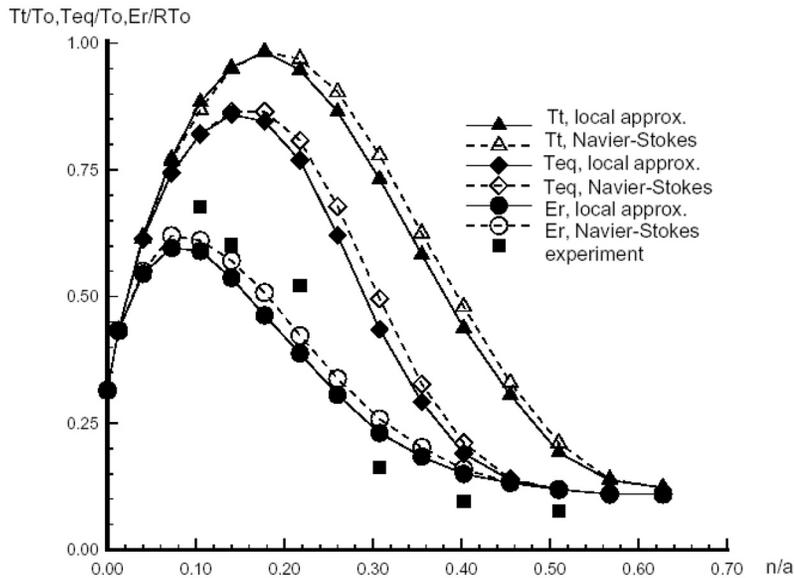


Fig. 1. Rotational energy E_r , translational T_t and equilibrium T_{eq} temperatures at critical stagnation line of the sphere. Experimental data from [10].

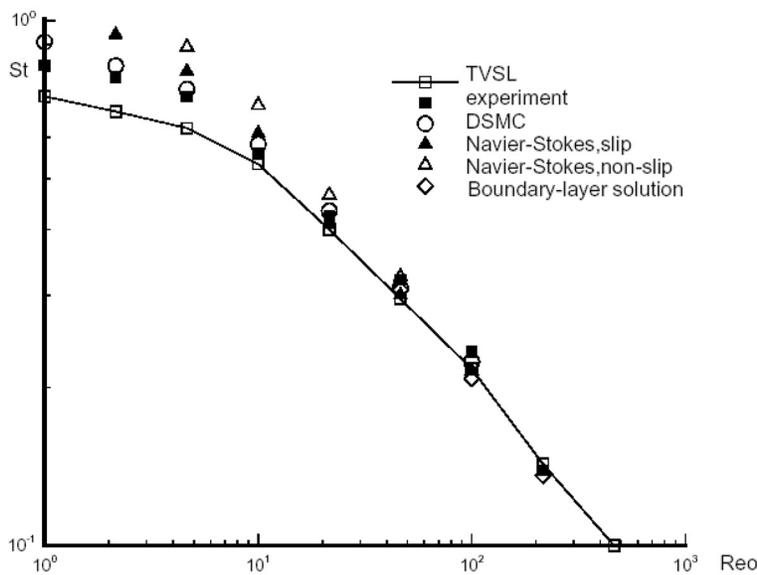


Fig. 2. Stanton numbers St on a sphere vs. Reynolds numbers Re_o for different medium models at various wind-tunnel conditions [11].

the amount of computational time. The applicability of such approach [3] is studied here by comparing the approximate solutions with the calculations of the full Navier-Stokes equations for nonequilibrium viscous flows.

The Navier-Stokes equations [9] written in coordinates s, n (s is the coordinate measured along the body

generatrix and n is the normal to the body surface) were modified by the following expressions [4]:

$$\rho = \rho_1(n); \quad \nu = \nu_1(n) \cos(s) \tag{1}$$

$$u = u_1(n) \sin(s) \tag{2}$$

$$T = T_1(n) + 0.5T_2(n) \sin^2(s) \tag{3}$$

$$E = E_1(n) + 0.5E_2(n) \sin^2(s) \tag{4}$$

$$p = p_1(n) + 0.5p_2(n) \sin^2(s) \tag{5}$$

$$p_1 = \rho_1 AT_1; p_2 = \rho_1 AT_2 \tag{6}$$

$$\mu = \mu_1(n) + 0.5\mu_2(n) \sin^2(s) \tag{7}$$

$$\mu_1 = T_1^\omega; \mu_2 = \omega\mu_1 T_2/T_1 \tag{8}$$

Substituting Eqs. (1)–(8) in the Navier-Stokes equations [9] at $s = 0$, we get the equations for ρ_1, ν_1, T_1, E_1 [3]. The equation terms should be considered as functions of the variable s and differentiated. Then substituting Eqs. (1)–(8) in the arrived equations, we get the equations for u_1, T_2, E_2 [3].

The one-dimensional implicit scheme [9] was used for solving the ‘local’ differential equations. The results calculated by approximation method (solid lines) and the Navier-Stokes equations (dashed lines) for streamlining a sphere by nitrogen at $Re_o = 14.4, M_\infty = 6.5$, and $t_w = 0.3$ are shown in Fig. 1. The comparison demonstrates that the local approximation technique, based on transformation (1)–(8), is applicable for the description of the nonequilibrium viscous flow.

4. Approximation of the thin viscous shock layer

The thin-viscous-shock-layer (TVSL) approximation [6] is used for analyzing nonequilibrium flows near a

blunt body. The TVSL equations are found from asymptotic analysis of the Navier-Stokes equations [9] at $\varepsilon \rightarrow 0, Re_o \rightarrow \infty$, and $\varepsilon Re_o = \text{const}$, where $\varepsilon = (\gamma-1)/(2\gamma)$ and γ is a specific heat ratio. The generalized Rankine-Hugoniot boundary conditions were formulated in [6,7]. The two-point matrix box-scheme [6] and Newton-Raphson method [7] were used for solving the grid equations.

The Stanton numbers St at the spherical stagnation point were calculated under wind-tunnel conditions at $M_\infty = 15, t_w = 0.15$, and various Reynolds numbers. The TVSL-model results (line) and solutions of the Navier-Stokes equations with slip boundary conditions (filled triangles) are shown in Fig. 2. At $Re_o > 10$, the results correlate well with experimental data [11] and numerical DSMC data [12].

5. Applications of ‘parabolized’ Navier-Stokes equations

Another marching method of solving simplified Navier-Stokes equations [1,4,5] allows economic usage of the computer resources. The ‘parabolized’ equations are found from the Navier-Stokes equations by excluding derivatives from viscous terms in the marching direction [5]. The regularization procedure [5] introduces the vector E^P as the portion of the streamwise flux [9] responsible for preserving ellipticity in the equations through the subsonic layer. Two approximations for

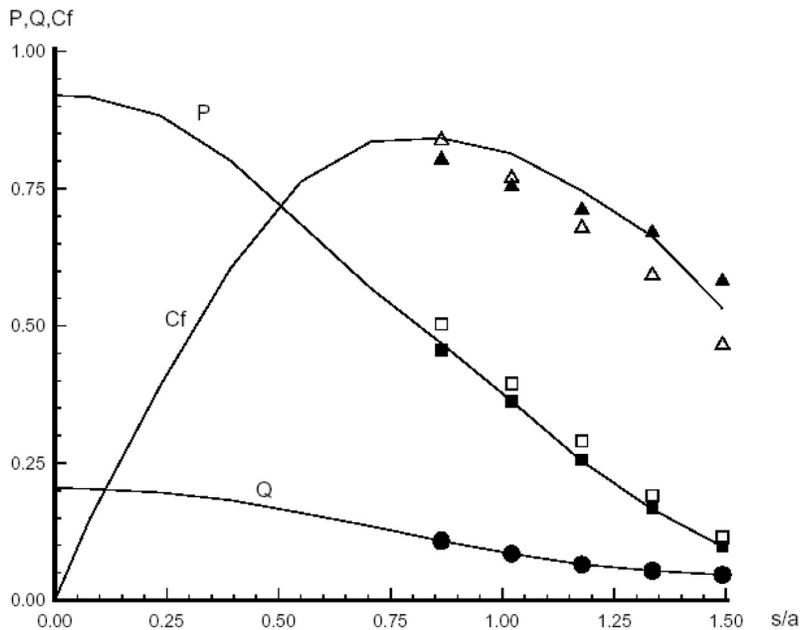


Fig. 3. Pressure P , heat flux Q , and friction C_f coefficients on sphere as the solutions of ‘parabolized’ (markers) and full (solid lines) Navier-Stokes equations.

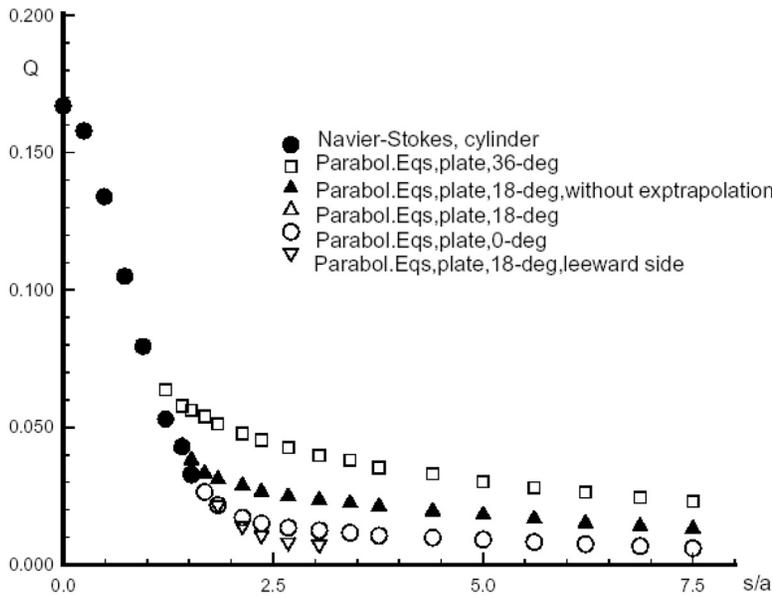


Fig. 4. Heat flux Q on the plate surface.

$\partial \mathbf{E}^p / \partial s$ are considered: (A) it is equal zero, and (B) it is downstream extrapolated. Seidel's method [9] was used for solving the 'parabolized' Navier-Stokes equations.

The problem of streamlining of the sphere by air at $Re_0 = 14.4$, $M_\infty = 6.6$, and $t_w = 0.34$ was tested. The distributions of pressure $P = p / \rho_\infty u_\infty^2$, heat flux $Q = q / \rho_\infty u_\infty^3$, and friction coefficient C_f along the spherical surface are presented in Fig. 3. The numerical solutions of the Navier-Stokes equations (solid lines) are compared with the solutions of the 'parabolized' equations that are presented by open markers (case A) and filled markers (case B). The comparison demonstrates that data obtained from different models favorably agree with each other. The usage of pressure-gradient downstream extrapolation (case B) offers the results that are the closest to the solutions of the Navier-Stokes equations. The calculation time for 'parabolized' equations is approximately five times less than that for the Navier-Stokes equations.

The 'parabolized' equations were used in computations of the flow near the plate with cylindrical blunt at the upstream flow parameters mentioned above. The zone of blunting was calculated using the Navier-Stokes equations, and the flowfield below the conjugate point was calculated using the simplified method. The results of calculating heat flux Q along the plate with cylindrical blunt at angles of attack $\alpha = 0^\circ$, 18° , and 36° are presented in Fig. 4 for the flat forward surface by filled triangles (case A) and open symbols (case B). The results for the leeward side at $\alpha = 18^\circ$ are presented by invert triangles. The selection of the regularization methods has insignificant influence on the heat flux values.

6. Conclusions

The study confirms the hypothesis of applicability of the Navier-Stokes equations, local approximation equations, 'parabolized' equations, and the thin-viscous-shock-layer approximation for the description of nonequilibrium flows near the simple-shaped bodies at Reynolds numbers $Re_0 > 10$. The regularization procedures influence significantly the approximate solutions.

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