Discontinuous Galerkin spectral element simulation of a type of wave propagation with large source terms

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Abstract

In this paper we simulate 1D quench propagation in superconducting magnets using cable-in-conduit conductors (CICC) by a discontinuous Galerkin (DG) spectral element method (SEM) and explicit Runge-Kutta time integration. The work seeks an algorithm exhibiting both high accuracy and efficiency. The supercritical helium flow is considered in the modeling of quench propagation in CICC, which can be expressed by the Euler equations with additional friction and coupled heat transfer between helium and conductor and conduit. Roe's approximate Riemann solver for a real gas/fluid is used to compute numerical flux and non-reflecting boundary condition is introduced in the algorithm. The method used here is highly parallelizable. Some numerical results are given and compared with those obtained by other simulation methods and experimental data.

Keywords: Quench propagation; Discontinuous Galerkin; Spectral element method

1. Introduction

In large-scale superconducting magnets systems, thermal stability is a key issue, and quench propagation is always an important consideration. Superconductors are designed to operate at very high current density, so when an external perturbation is strong enough, the superconductor will go from the superconducting state to the normal state (resistive conductor), this transition constitutes a quench. When the magnet quenches, the normal zone evolves and expands with time, it will encompass regions at liquid helium temperature at its periphery to a maximum at the quench initiating point. We focus on cable-in-conduit conductors (CICC) which are commonly used in large-scale magnets and as the name implies, consist of a superconducting cable inside a metal pipe containing the liquid helium coolant. The liquid helium is under supercritical regime. The fluid flow in the tube of the CICC is very complicated, and is governed by the unsteady convection-diffusion equations in the regime of high Reynolds number and low Mach number. One difficulty is the complicated coupling heat transfer between fluid and conductors, and the conduit wall. Another difficulty relates to the highly non-linear physical properties of solid materials and liquid helium [1].

Many methods have been considered for quench simulation in superconducting magnets, for example, finite element, finite volume, and finite difference methods [2,3,4,5,6,7,8,9]. However, most of them are first-order or second-order methods. For long time wave traveling problems, large dissipation and dispersion will add to the numerical solutions by using low-order methods. The main goal of our research is to seek an algorithm exhibiting both high accuracy and efficiency in solving this particular problem. In this paper we focus on 1D discontinuous Galerkin (DG) spectral element method (SEM) and explicit Runge-Kutta time integration method [10]. The SEM is a weighted-residual technique for the solution of partial differential equations that combines the geometric flexibility of low-order finite element methods with the rapid convergence rate of spectral methods. Roe's approximate Riemann solver

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is used to treat the numerical flux generated in the weak form of the governing system of equations [11,12], which is highly efficient without adding numerical viscosities. This method is highly parallelizable.

2. Modeling of quench propagation in CICC

The typical CICC has a large ratio of length-to-diameter, up to 10^5, the compressible helium flow can be simplified by considering it only along the longitudinal direction of the channel [13]. The cable of the CICCs can be assumed to be at uniform temperature across the conductor. The helium exchanges heat with conductors and conduit wall. Friction also plays an important role in the helium flow. The basic governing equations consist of continuity, momentum and energy for the helium, and the coupled heat balance equations for conductors and conduit. We can write down the system in the conservative form:

$$\frac{\partial}{\partial t}Q+\frac{\partial}{\partial x}F=S \tag{1}$$

$$\rho_{st}C_{st}\frac{\partial T_{st}}{\partial t}=\frac{\partial}{\partial x}\left(k_{st}\frac{\partial T_{st}}{\partial x}\right)-\frac{h_{st}P_{st}}{A_{st}}(T_{st}-T)-\frac{h_{st}P_{st}}{A_{st}}(T_{st}-T_{jk})+J(x,t,T_{st})+\alpha \tag{2}$$

$$\rho_{jk}C_{jk}\frac{\partial T_{jk}}{\partial t}=\frac{\partial}{\partial x}\left(k_{jk}\frac{\partial T_{jk}}{\partial x}\right)-\frac{h_{jk}P_{jk}}{A_{jk}}(T_{jk}-T)-\frac{h_{st}P_{st}}{A_{jk}}(T_{jk}-T_{st}) \tag{3}$$

where

$$Q=\begin{pmatrix}\rho\\ m\\ \varepsilon\end{pmatrix},\ F=\begin{pmatrix}m\\ p+m^2/\rho\\ (\varepsilon+p)m/\rho\end{pmatrix},\ S=\begin{pmatrix}0\\ s_1\\ s_2\end{pmatrix},\ m=\rho v,$$

$$\varepsilon=\rho\left(e+v^2/2\right)=\rho E$$

$$S_1=-\frac{f\rho v|v|}{2d_h},\ s_2=\frac{h_{st}P_{st}}{A_{He}}(T_{st}-T)+\frac{h_{jk}P_{jk}}{A_{He}}(T_{jk}-T)$$

and $J(x,t,T_{st})=\rho_e\left(\frac{I}{A_{Cu}}\right)^2$ is the Joule heating in the conductors. T, T_{st}, T_{jk} are the temperature of helium, conductors and the conduit, respectively. p_{st}, p_{jk} are the wet perimeters of conductors and conduit, respectively. h_{st}, h_{jk} are the heat transfer coefficients between helium with conductors and with the conduit, respectively. ρ_{st}, ρ_{jk}, ρ_e are the density of conductors and the conduit, and the resistivity of copper, respectively. c_{st}, c_{jk} are the specific heat of conductors and conduit, respectively. k is the thermal conductivity, α is the external heat source beside the Joule heating. f is the Darcy friction factor for internal channel flow. The equation of state of helium is added to close the system:

$$p=p(\rho,T) \tag{4}$$

Equations (1)–(4) with well-defined boundary conditions and initial condition, consist of the governing system of 1D quench propagation problems.

The left-hand side of Eq. (1) is the Euler equation, so it is reasonable to analyze helium flow by characteristics methods. Three Riemann variables are obtained by integration of the Euler equations. The integral in the Riemann variables is approximated by curve fitting to the helium data output from HEPAK [14]. An approximate polynomial is used based on a carefully chosen equilibrium state (p_0,T_0) at each time step to solve the Riemann integral with global second order [15]:

$$\frac{1}{\rho c}=\left(\frac{1}{\rho c}\right)_0+a_1(p-p_0)+a_2(p-p_0)^2+a_3(T-T_0)+a_4(T-T_0)^2+a_5(p-p_0)(T-T_0) \tag{5}$$

This is very important to successfully implement DG spectral element methods.

3. Discontinuous Galerkin spectral element methods

We can discretize the system of Eqs. (1)–(3) by using discontinuous Galerkin spectral element methods. In one spatial dimension, the region under consideration is divided into non-overlapping elements. Each element is mapped individually onto the $[-1,1]$ domain by an isoparametric transformation:

$$x=x_{k-1}+\frac{1+\xi}{2}(x_k-x_{k-1})=x_{k-1}+\frac{1+\xi}{2}d^k$$

The system of Eq. (1) becomes:

$$\frac{\partial}{\partial t}\bar{Q}(\xi,t)+\frac{\partial}{\partial\xi}\bar{F}(\xi,t)=\bar{S}(\xi,t)$$

where $\bar{Q}(\xi,t)=Q(x^k(\xi),t)$, similarly for $\bar{F}(\xi,t)$ and $\bar{S}(\xi,t)$. The discontinuous Galerkin version of the spectral element method approximates the solution and the fluxes by the Nth-order polynomials that are defined at the Legendre-Gauss quadrature points [16]:

$$\bar{Q}(\xi)=\sum_{j=0}^{N}\bar{Q}_j\ell_j,\ \bar{F}(\xi)=\sum_{j=0}^{N}\bar{F}_j\ell_j$$

No assumptions are made about the continuity of the solutions, and the residual is required to be orthogonal to the approximation space within each element with collocation form. Transform the system (1) to the weak form, integrate by part, replace the integral by Gauss

quadratures, and, after some algebraic manipulation, we have the final approximation in 1D quench propagation problems with the collation form,

$$\frac{d\bar{Q}_i}{dt} + \left[\bar{F}(1)\frac{\ell_i(1)}{w_i} - \bar{F}(-1)\frac{\ell_i(-1)}{w_i} - \sum_j \bar{F}_j \frac{(\ell_i', \ell_j)_N}{w_i}\right] = \bar{S}_i \tag{6}$$

where the discrete inner product is the Gauss quadrature

$$(u,v)_N = \sum_{i=0}^{N} u_i v_i w_i, \quad i = 1,2,\cdots N \tag{7}$$

Similarly, the DG spectral element discretization can be given to the heat balance equations by the additional equation to apply the second order term, i.e.

$$\theta - \frac{\partial(\gamma T_{st})}{\partial x} = 0$$

where $\gamma = \frac{k_{st}}{\rho_{st} c_{st}}$ is the equivalent diffusion coefficient of conductor. The spectral discretization is given by

$$\bar{\theta}_i + \left[(\gamma T_{st})|_{\xi=1}\frac{\ell_i(1)}{w_i} - (\gamma T_{st})|_{\xi=-1}\frac{\ell_i(-1)}{w_i} + \sum_j (\gamma T_{st})_j \frac{(\ell_i', \ell_j)_N}{w_i}\right] = 0 \tag{8}$$

Roe's approximate Riemann solver is chosen to compute the numerical flux at the interface [12]. Since no analytic formulae are available for the equation of state (EOS) for helium, the first derivatives in the transform Jacobian are solved by cubic spline techniques [17].

The use of the Riemann solver at the element faces makes the imposition of boundary conditions simple. Boundary conditions are implemented by specifying the external state as the input for the Riemann solver.

The semi-discrete approximation system (6) and (8) can be integrated by explicit schemes. We apply the fourth-order Runge-Kutta method to integrate the linear system obtained. The coefficients for the fourth-order Runge-Kutta method can be found in [17].

For unsteady problems the time step is restricted by the CFL condition and by the second order derivatives in the heat balance equations in the conductors and conduit. So to keep the calculation stable and reasonably accurate in time, the time step may not surpass the stability region of the explicit fourth-order Runge-Kutta method.

4. Numerical results and discussion

This method is tested on two cases. The first one was reported by Arp [2], in which a short NbTi superconductor coil was used with a large external heat pulse to quench it. In this case we assumed subsonic velocity at the exit and symmetric condition in the middle point. A Gaussian heat pulse (for numerical stability reasons) is imposed in the middle to initiate the quench simulation. Figures 1 and 2 show helium induced-flow velocity obtained by using DG-spectral element methods and a commercial FEM code (GANDALF). DG-SEM got a better resolution and higher stability than GANDALF. Because DG-spectral element methods applied conservative form of the governing equations, imposed the

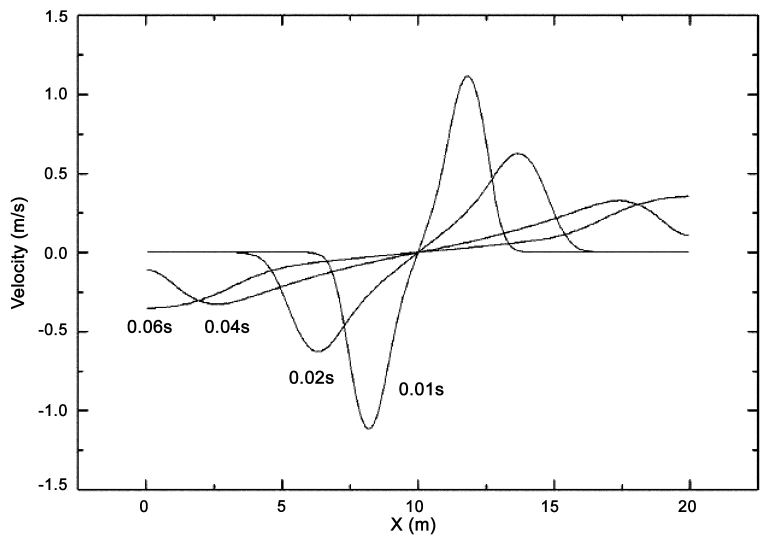

Fig. 1. Distribution of helium induced-flow velocity using DG-SEM with an external heat pulse 8×10^3 W/m in 1 m zone for 0.001 s.

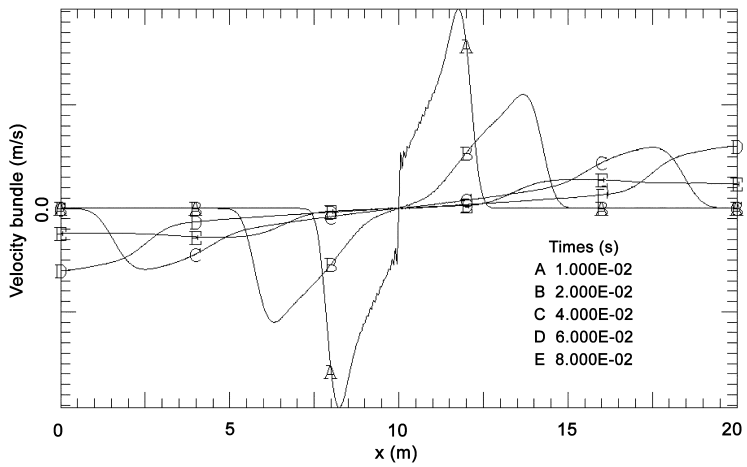

Fig. 2. Distribution of helium induced-flow velocity using GANDALF code with an external heat pulse 8×10^3 W/m in 1 m zone for 0.001 s.

numerical flux balance across the interfaces of each element, the numerical solution is very stable against large non-linear disturbances.

The second benchmark experiment was chosen from Ando et al. [18], also consisting of a short NbTi superconductor coil with a much larger sharp heat pulse to lead to quench. The numerical results by DG-SEM and GANDALF code were compared against the same experimental data [18]. The evolution and distribution of quench pressures are shown in Figs. 3–4. There is very good agreement between the two numerical methods. The difference in maximum quench pressure is about 3~5% between DG-SEM and GANDALF. The predicted exit velocity of helium is about the same for both codes. However, the simulation results obtained by GANDALF code exhibited obvious oscillations that do not exist in DG-SEM.

Figure 5 illustrates very good agreement for the simulation of normal zone propagation. Considering the factors in experimental measurement and the correction formulae approximation used in the mathematical models, these results are quite good, and acceptable for engineering purposes.

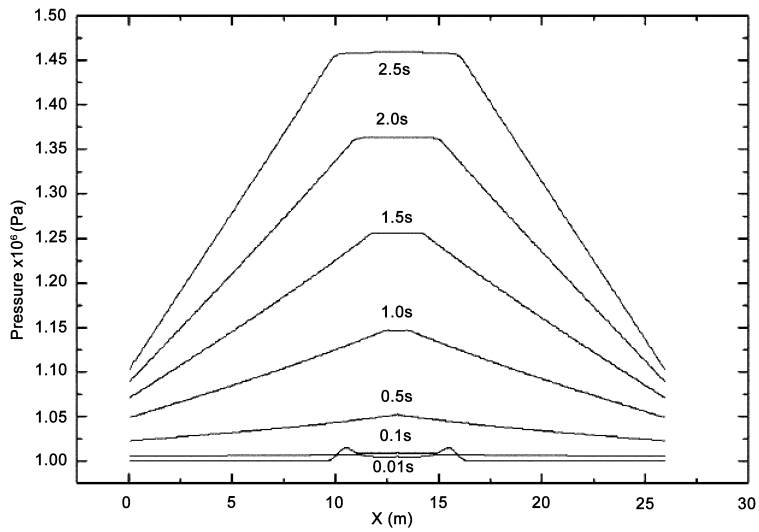

Fig. 3. Evolution and distribution of quench pressure in CICC using DG-SEM with $I = 1.5$ kA in Ando's case.

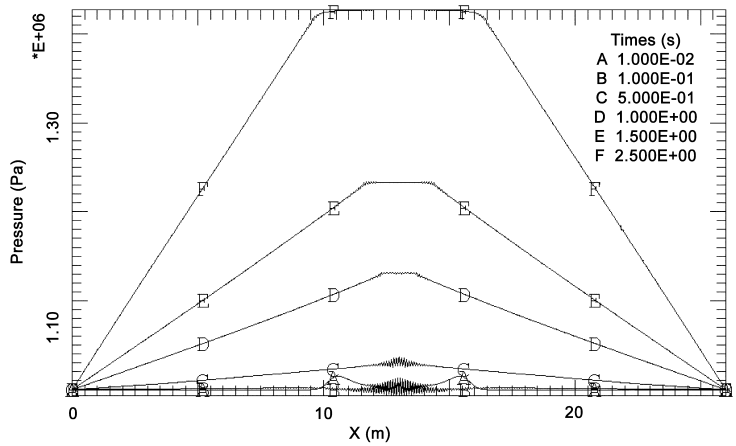

Fig. 4. Evolution and distribution of quench pressure in CICC using GANDALF code with $I = 1.5$ kA in Ando's case.

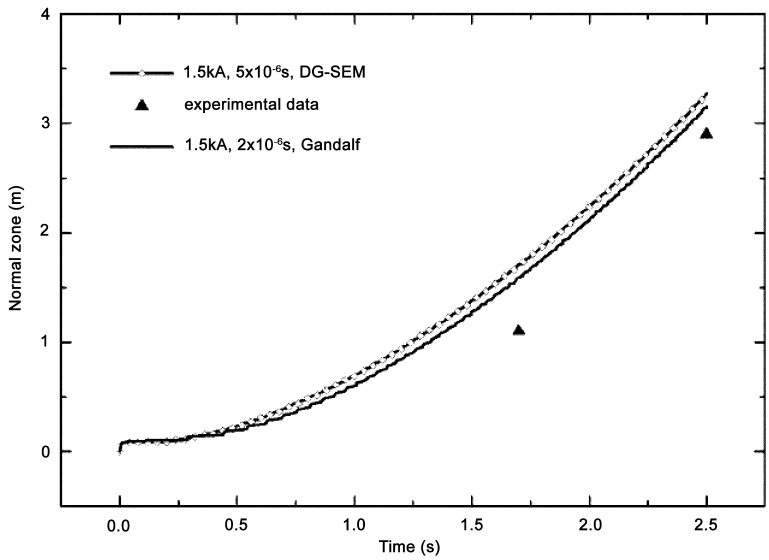

Fig. 5. Comparison of evolution of normal zone ($I = 1.5$ kA) in Ando's case.

5. Conclusions

Discontinuous Galerkin spectral element methods were successfully used to track a special moving front problem, quench propagation in CICCs. Special effort was placed into dealing with the real gas/fluid properties (supercritical fluid). DG-SEM has the advantage of high resolution in a large gradient region of solution. DG-SEM is designed for parallel computation without introducing extra expense. Parallelization of the scheme is the natural next step in this research.

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