Repair of delaminated beams via piezoelectric patches

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Abstract

This research presents the application of piezoelectric patches on the repair of delaminated beams subjected to concentrated static loading. The voltage applied on the piezoelectric patches is calculated and designed to erase the shear stress singularity at the tips of the delamination so that the delaminated beam can be repaired in the sense that the sliding mode of fracture on the beam can be removed. The dependence of the voltages on the location and the size of the delamination is carefully studied.

Keywords: Sliding mode of fracture; Piezoelectric materials; Repair of structures; Delaminated beams

1. Introduction

Damage in aerospace, aeronautical, mechanical, and civil and offshore structures often result from reasons such as fatigue, corrosion, or accidents. Most conventional repair methods involve welding, riveting or mounting additional patches to the parent structure without removing the damaged portion [1,2,3]. These methods often only restore the lost stiffness under a unique external loading and are therefore passive and inflexible for the case where external loading changes.

The advent of smart materials, such as piezoelectrics and shape memory alloys, opens up new opportunities for improved repair techniques to overcome some of the limitations of conventional repair methods [4].

Composite materials have been receiving increased attention in recent years, especially for aeronautical and marine applications. Delamination in composites has been a topic of concern in such applications. This has been extensively reviewed by [5]. Effective repair of delaminated structures, however, still needs to be comprehensively investigated.

The objective of this paper is to present a methodology for the repair of delaminated beams applying piezoelectric materials.

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2. Model of repair of delaminated beams via piezoelectric patches

A delaminated beam, as shown in Fig. 1, subjected to a static loading is to be repaired by the use of piezoelectric patches. The material and geometric parameters of the delaminated beam are denoted as E for the Young's modulus of the host beam; H the thickness of the beam; and T the width of the beam. The thickness of the upper layer of the delamination is t and the length of the delamination is a. Since the mid-planes of the two layers are off the mid-plane of the delaminated beam, axial elongation and compression on the two layers will thus be induced due to the bending of the beam. It is assumed the tensile force Δp_1 and compressive force Δp_2 are induced on the top and bottom layer of the delamination. Due to the continuity of the deflection at the left tip and right tip of the delamination based on Euler–Bernoulli beam theory, we have:

$$u_{1L} - \frac{1}{2} w'_L\big|_{x=x_l}(H-t) = u_{0L} \tag{1a}$$

$$u_{2L} - \left(-\frac{t}{2} w'_L\big|_{x=x_l}\right) = u_{0L} \tag{1b}$$

$$u_{1R} - \frac{1}{2} w'_R\big|_{x=x_r}(H-t) = u_{0R} \tag{2a}$$

$$u_{2R} - \left(-\frac{t}{2} w'_R\big|_{x=x_r}\right) = u_{0R} \tag{2b}$$

where w_L is the flexural deflection of the beam element at the left side of the delamination; x_l is the coordinate of

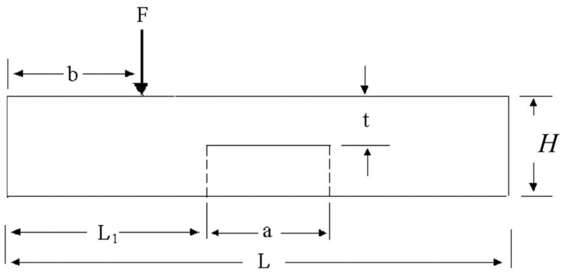

Fig. 1. Delaminated cantilever beam.

the left tip of the delamination; prime indicates the derivative with respect of x; u_{1L} and u_{2L} indicate the horizontal deflection of the upper and lower layers of the delamination at the left tip; u_{0L} is the horizontal deflection of the mid-plane of the delaminated beam at the left tip of the beam. The subscript R stands for the variables at the right end of delamination accordingly.

The elongation and compression of the mid-plane of the two layers can be respectively expressed as:

$$u_{1R} - u_{1L} = \frac{\Delta p_1 a}{EtT} \tag{3a}$$

$$u_{2R} - u_{2L} = -\frac{\Delta p_2 a}{E(H-t)T} \tag{3b}$$

The characteristic of the non-deformable mid-plane of the delaminated beam implies $U_{0R} - U_{0L} = 0$. Comparison of Eqs. (3a,b) and Eqs. (1a,b) and (2a,b) leads to:

$$\frac{\Delta p_1 a}{EtT} + \frac{\Delta p_2 a}{ET(H-t)} = -\frac{H}{2}\left(w'_L\big|_{x=x_l} - w'_R\big|_{x=x_r}\right) \tag{4a}$$

$$\Delta p_1 = \Delta p_2 \tag{4b}$$

and finally we have:

$$\Delta p_1 = \Delta p_2 = -\frac{ETH\beta}{2a}\left(w'_L\big|_{x=x_l} - w'_R\big|_{x=x_r}\right) \tag{5}$$

where $\beta = \left(\frac{1}{t} + \frac{1}{(H-t)}\right)^{-1}$.

From the above mechanics analysis it is shown that, at the tip of the delamination, tensile and compressive forces are induced at the upper and lower part of the tip due to the bending of the beam. The induced tensile and compressive forces will lead to the discontinuity of the shear force at the tip of the delamination, which will definitely result in the shear force singularity and therefore lead to the sliding mode of fracture at the tip of the delamination. Piezoelectric patches will be applied to induce shear force at the interface of the piezoelectric layer and the host delaminated beam by applying a suitable voltage so that the sliding fracture mode at the tip of the delamination can be controlled and hence the beam can be repaired accordingly.

The first pair of piezoelectric actuators is to be bonded with their right ends in the vicinity of the left tip of the delamination, while the second pair is to be bonded with their left ends in the vicinity of the right tip of the delamination. It is expected that each pair of the piezoelectric layers will induce the tensile and compressive forces whose magnitude are Δp_1 given in Eq. (5) at the interfaces of the piezoelectric patches and the host beam.

Crawley and de Luis [6] had proposed the following expression for the shear force between a metal substrate and the piezoelectric layer under the assumption of complete bonding between them:

$$P = \frac{EHT}{\Psi + 6}\Lambda \tag{6}$$

where, $\Psi = EH / E_a h_1$, h_1 is the thickness of the piezoelectric patch, $\alpha = 6$ when a bending bar is considered, and $\Lambda = d_{31}V / h_1$. E_a is the equivalent Young's modulus of the piezoelectric layer for one-dimensional problems and d_{31} is piezoelectric charge coefficient.

Comparing Eq. (5) and Eq. (6) and considering $P = \Delta p_1$ lead to the expression of the voltages applied to the pairs of the piezoelectric patches as follows:

$$V = -\frac{h_1\beta(\Psi + 6)}{2ad_{31}}\left(w'_L\big|_{x=x_l} - w'_R\big|_{x=x_r}\right) \tag{7}$$

The same voltages will be applied at the two pairs of the piezoelectric layers. For each of the piezoelectric pairs, the voltage at the upper and lower piezoelectric layers should be applied with the same amount but with different alignment of the poling direction of the piezoelectric patch so that the forces in Fig. 3 can be induced.

In the next session, the derivation of the voltages applied on the piezoelectric patches will be provided for the repair of delaminated beams.

3. Solution for repair of delaminated beams

A sectional analysis is necessary for the derivation of the response of delaminated beams under static loading with different boundary conditions. We will only briefly list the results of the voltage applied on the piezoelectric layers bonded on the host delaminated beam in Fig. 2 as follows.

The solution for the voltages will be provided for simply supported and cantilever delaminated beam structures for three cases: $b < L_1$, $L_1 < b < L_1 + a$, and $b > L_1 + a$. The responses of slope in these beams

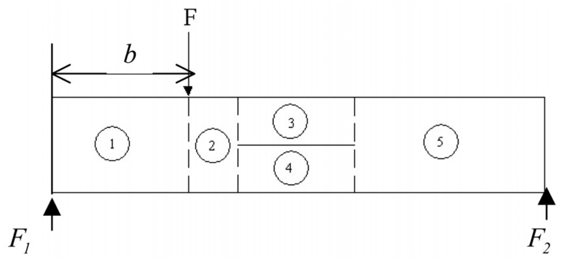

Fig. 2. Sectional analysis of a delaminated beam.

are referenced from the book by Gere [7], and the solutions for the voltage are listed as follows:

- Simply supported beam
- $b \geq L_1$:

$$w'_L\big|_{x=x_l} - w'_R\big|_{x=x_r} = -\frac{Pb}{LEI}(a(L-L_1) - a^2/2) \quad (8a)$$

$$V = \frac{Pbh_1\beta(\Psi+6)}{2Ld_{31}EI}((L-L_1) - a/2) \quad (8b)$$

- $L_1 < b \leq L_1 + a$

$$w'_L\big|_{x=x_l} - w'_R\big|_{x=x_r} = -\frac{P(L-b)}{6LEI}(2Lb - b^2 - 3L_1^2) - \frac{Pb}{6LEI}(L^2 - b^2 - 3(L-a-L_1)^2 \quad (9a)$$

$$V = \frac{Ph_1\beta(\Psi+6)}{12Lad_{31}EI}\left((L-b)(2Lb - b^2 - 3L_1^2) + b(L^2 - b^2 - 3(L-a-L_1)^2\right) \quad (9b)$$

- $b > L_1 + a$:

$$w'_L\big|_{x=x_l} - w'_R\big|_{x=x_r} = -\frac{P(L-b)}{LEI}(aL_1 + a^2/2) \quad (10a)$$

$$V = \frac{P(L-b)h_1\beta(\Psi+6)}{2Ld_{31}EI}(L_1 + a/2) \quad (10b)$$

- Cantilever beam
- $b \leq L_1$:

$$w'_L\big|_{x=x_l} - w'_R\big|_{x=x_r} = 0 \quad (11a)$$

$$V = 0 \quad (11b)$$

- $L_1 < b \leq L_1 + a$:

$$w'_L\big|_{x=x_l} - w'_R\big|_{x=x_r} = -\frac{P}{2EI}(2bL_1 - L_1^2 + b^2) \quad (12a)$$

$$V = \frac{Ph_1\beta(\Psi+6)}{4ad_{31}EI}\left(2bL_1 - L_1^2 + b^2\right) \quad (12b)$$

- $b > L_1 + a$:

$$w'_L\big|_{x=x_l} - w'_R\big|_{x=x_r} = -\frac{P}{2EI}(-2ba + a^2 + 2aL_1) \quad (13a)$$

$$V = \frac{Ph_1\beta(\Psi+6)}{4d_{31}EI}(-2b + a + 2L_1) \quad (13b)$$

4. Numerical simulations

Numerical simulations are conducted to study the repair of delaminated simply supported and cantilever beams. The parameters for the beams, piezoelectric patches and bonding layers considered are as follows: $L = 1m$, $h_1 = 0.0005m$, $H = 0.05m$, $F / T = 1ooN / m$, $E = 210 \times 10^9\ N / m^2$, $E_p = 63 \times 10^9\ N / m^2$, $d_{31} = 190 \times 10^{-12}\ m / V$.

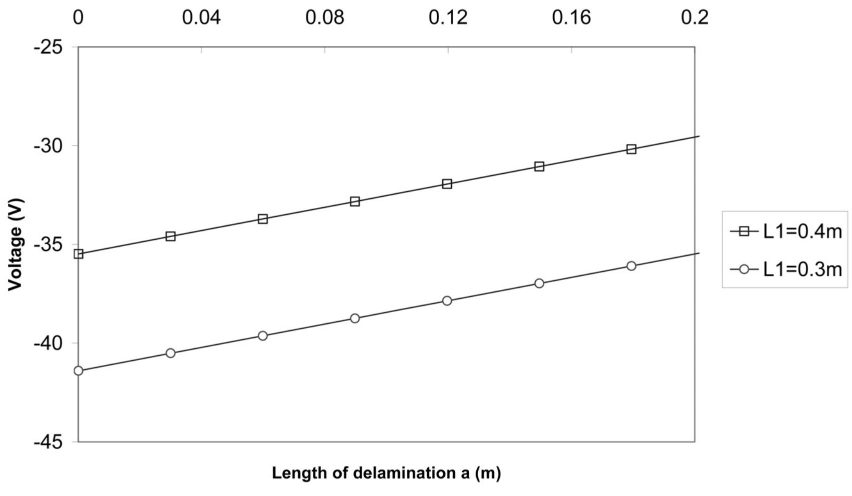

Fig. 3. Voltage applied to the piezoelectric patches versus the size of the delamination.

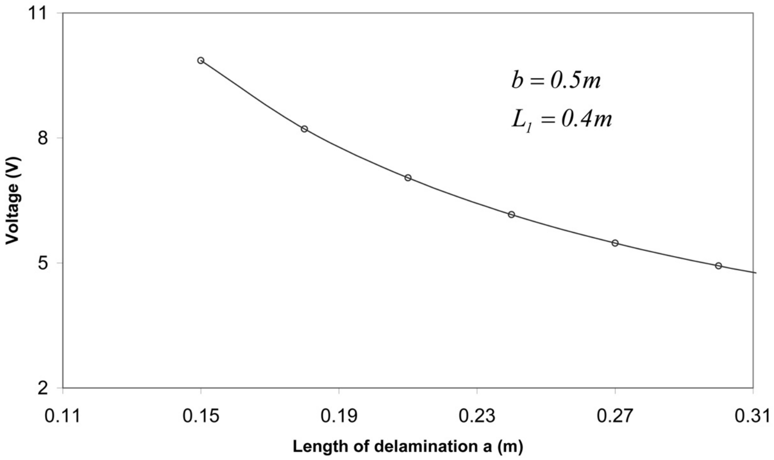

Fig. 4. Voltages applied to the piezoelectric layer versus the size of the delamination.

The effects of the location and size of the delamination on the required voltages applied to the piezoelectric patches are presented based on numerical simulations for the above simply supported beam and cantilever beam. Only one case will be studied, namely, $b < L_1$, for simply supported beam and $L_1 + a > b > L_1$ for cantilever beam. For the other cases, the results are not included in the short paper, but the conclusions will be included in the next section.

The voltage is plotted in Fig. 3 at $b = 0.2m$, and $L_1 = 0.3m, 0.4m$ for a simply supported delaminated beam. The results show that the magnitude of the voltage is lower if the delamination is farther from the force. Another observation is that the voltage becomes smaller with increase in the size of the delamination. These two observations can be explained from Eq. (8b).

As for the cantilever beam, results are shown in Fig. 4, which shows that that lower voltage is needed for a bigger size of the delamination when the loading position is $b = 0.5m$ and $L_1 = 0.4m$. This observation can be explained from Eq. (9b) from which we can see $V \infty 1 / a$.

5. Conclusions

The first result from the research is that the higher voltage is needed for the repair of delaminated beams when the delamination is located towards the center of the host beam in thickness direction. Second, higher voltage is required for shorter delamination both for simply supported beams and cantilever beams, except when the loading is at the right side of the delamination in simply supported beams. Finally, when the loading is not applied on the delamination, lower voltage is necessitated if the delamination is further from the loading location in delaminated simply supported beams.

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