Interval finite element analysis of large structures with uncertain parameters

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Abstract

This paper uses a non-probabilistic interval finite element method (IFEM) for the eigenvalue and frequency response function (FRF) analysis of a structure with uncertain parameters. The comparison of the obtained results with the results of Monte Carlo simulations proves that the IFEM gives a good approximation of the dynamic behaviour of uncertain structures. The use of a component mode synthesis (CMS) method to divide large structures into deterministic components and a non-deterministic residual structure can reduce the computation time of the IFEM without loss of accuracy.

Keywords: Uncertainty; Interval finite element method; Eigenvalues; Frequency response function; Component mode synthesis; Substructuring

1. Introduction

Nowadays, the finite element method (FEM) has become an indispensable tool for the numerical optimisation and validation of structural designs. However, it is sometimes very difficult to define a reliable finite element (FE) model for these purposes, especially when a number of physical properties are uncertain.

In this paper, an interval finite element method (IFEM) is used to predict the influence of uncertain parameters for which no statistical data are available. An interval dynamic analysis of the Garteur benchmark aircraft with three uncertain parameters is performed with a hybrid implementation of the IFEM, developed by Moens [1,2]. A component mode synthesis (CMS) method is applied for the reduction of the numerical model. The interval results of this reduced model are compared with the results of the full model.

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2. Methodology

2.1. The interval finite element method for frequency response function analysis

The hybrid method for interval frequency response function (FRF) analysis, developed by Moens, is based on the deterministic modal superposition principle. For *undamped* structures, this principle states that, considering the first n_{modes} modes, the frequency response function between degrees of freedom (DOF*s*) j and k equals:

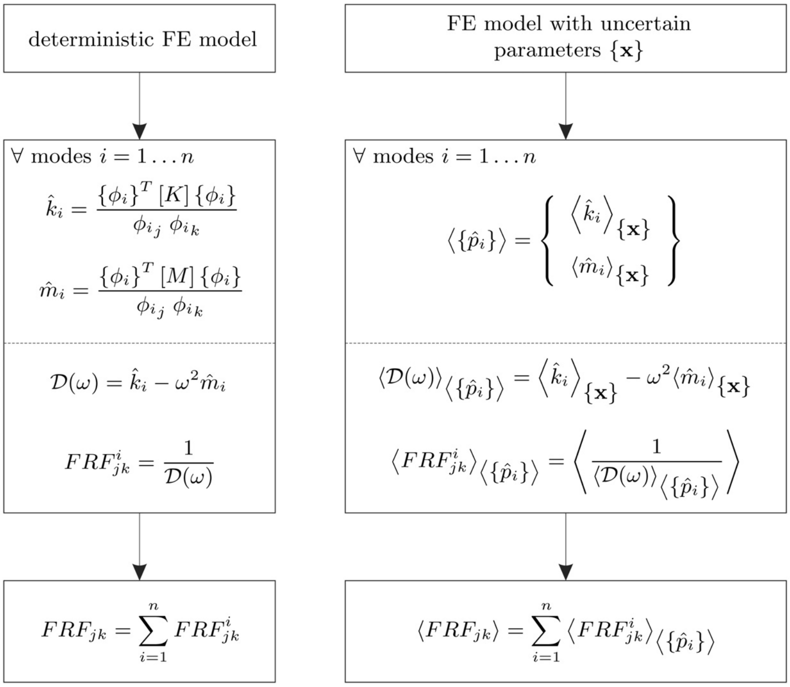
$$FRF_{jk} = \sum_{i=1}^{n_{modes}} \frac{\phi_{i_k}\phi_{i_j}}{\{\phi_i\}^T[K]\{\phi_i\} - \omega^2\{\phi_i\}^T[M]\{\phi_i\}} = \sum_{i=1}^{n_{modes}} \frac{1}{\hat{k}_i - \omega^2\hat{m}_i} \quad (1)$$

with $\hat{k}_i$ and $\hat{m}_i$ the modal parameters defined as:

$$\hat{k}_i = \frac{\{\phi_i\}^T[K]\{\phi_i\}}{\phi_{i_j}\phi_{i_k}}, \quad \hat{m}_i = \frac{\{\phi_i\}^T[M]\{\phi_i\}}{\phi_{i_j}\phi_{i_k}} \quad (2)$$

The function $\mathcal{D}(\omega) = (\hat{k}_i - \omega^2\hat{m}_i)$ expresses the modal response denominator as a function of frequency.

The deterministic modal superposition principle has been translated into an IFEM for FRF analysis (Fig. 1).



(a) Deterministic algorithm (b) Set translated algorithm

Fig. 1. Translation of the deterministic modal superposition algorithm to an equivalent IFE procedure.

The interval translation shows that the total envelope FRF can be calculated in three principal steps:

1. For all n_{modes} taken into account, the correct ranges of the modal parameters $<\hat{k}_i>$ and $<\hat{m}_i>$ are determined using a global minimisation and maximisation over the vector space $\{\mathbf{x}\}$ defined by the input interval parameters:

$$\left\langle \hat{k}_i \right\rangle_{\{\mathbf{x}\}} = \left[\min_{\{x\}\in\{\mathbf{x}\}} (\hat{k}_i(\{x\}), \max_{\{x\}\in\{\mathbf{x}\}} (\hat{k}_i(\{x\}) \right] \quad (3)$$

$$\langle \hat{m}_i \rangle_{\{\mathbf{x}\}} = \left[\min_{\{x\}\in\{\mathbf{x}\}} (\hat{m}_i(\{x\}), \max_{\{x\}\in\{\mathbf{x}\}} (\hat{m}_i(\{x\}) \right] \quad (4)$$

2. The modal envelope FRF is calculated by substituting the ranges of the modal parameters in the denominator function $\mathcal{D}(\omega)$ and subsequently inverting the resulting denominator function range.
3. The total interval FRF is obtained by the summation of the contribution of each mode.

The approximation of the modal envelope FRF can be improved substantially by taking the exact eigenfrequency ranges $<f_i>$ into account [2]. Therefore, an additional optimisation is performed in the first step of the interval algorithm:

$$\langle f_i \rangle_{\{\mathbf{x}\}} = \left[\min_{\{x\}\in\{\mathbf{x}\}} (f_i(\{x\}), \max_{\{x\}\in\{\mathbf{x}\}} (f_i(\{x\}) \right] \quad (5)$$

A similar interval procedure can be obtained for structures with *proportional damping* [1].

The efficiency of the first step of the interval finite element (IFE) procedure is primordial, as it requires six global optimisation procedures for each mode (Eqs (3–5)). For each goal function evaluation in the optimisation procedure, an FE eigenvalue analysis has to be performed, which can be very time-consuming for large numerical models. The use of a reduced model can substantially decrease the model evaluation time and hence increase the efficiency of the optimisation procedure drastically.

2.2. Component mode synthesis

The aim of the well-known CMS method [3,4] is to reduce the computational cost of large structures and to enable a solution strategy in which individual components can be optimised without the need of the recalculation of the total structure. Here, the CMS method is used to divide a structure into deterministic components on one hand and a non-deterministic residual structure on the other hand. All components are processed independently, resulting in a set of reduced matrices. These are then assembled with the non-reduced residual structure, to form the reduced stiffness and mass matrices [*K*] and [*M*] of the reduced model. Consequently, the uncertainties in the residual structure affect directly these stiffness and mass matrices. Hence, in the first step of the interval FRF algorithm, only the residual structure needs to be recalculated during the optimisation of the modal parameters $\hat{k}_i$, $\hat{m}_i$ and f_i, while the deterministic components remain unchanged. For large models, this can lead to a considerable reduction of the computation time of the IFEM.

3. Case study: the Garteur benchmark problem

3.1. Problem definition

The Garteur benchmark problem [5] consists of a small-scale, simplified aluminium aircraft model with a length of 1.5 m, a wing span of 2 m and a mass of 44 kg. The fuselage of the aircraft consists of a rectangular plate with a thickness of 50 mm. The tail with a thickness of 10 mm is connected rigidly to the fuselage. The wings are connected to the fuselage through an intermediate steel plate. Wingtips are connected rigidly at both ends of the wings. Both the wings and the wingtips are rectangular plates with a thickness of 10 mm. The FE model, as illustrated in Fig. 2, contains almost 20 000 degrees of freedom.

The Garteur aircraft model contains some inherent uncertainties, due to a lack of knowledge on the physical model as well as uncertainty on the modelling level. Three uncertainties are considered during the dynamic analysis of the structure:

1. A first source of uncertainty is the thickness of the visco-elastic layer, glued on to a part of the wings. The uncertainty on this thickness ranges between 0.1 and 1.6 mm, with a nominal value of 1.1 mm.
2. A second source of uncertainty is the stiffness of a part of the connection between wings and fuselage – an inherent modelling uncertainty. In the assembled model, this connection is modelled with an interconnecting plate parallel to the wings. The fuselage is connected rigidly to this plate. The DOFs of the interconnecting plate are connected rigidly to the wings, except for the DOFs on the edge of the plate. These DOFs are connected to the corresponding DOFs on the wings using linear springs. The dimension and the stiffness of the connection between the wings and the fuselage can then be varied in a continuous way by changing the stiffness of these springs. In the performed analyses, this stiffness ranges between 10 and 10^{15} N/m, with a nominal value of 10^8 N/m.
3. A third uncertainty is introduced on the Young's modulus of the wing material, with a range between 67.5 and 68.5 GPa, with a nominal value of 68.0 GPa.

3.2 Proportionally damped interval frequency response function analysis

The influence of the three uncertain parameters on the proportionally damped interval FRF between the wingtips of the aircraft model is investigated. The input and output DOFs are indicated in Fig. 2. For the calculation of the interval FRF, 14 modes are taken into account, covering a frequency range up to 160 Hz. The damping ratios of all considered modes have values between 1.0 and 2.8%. A full model as well as a model

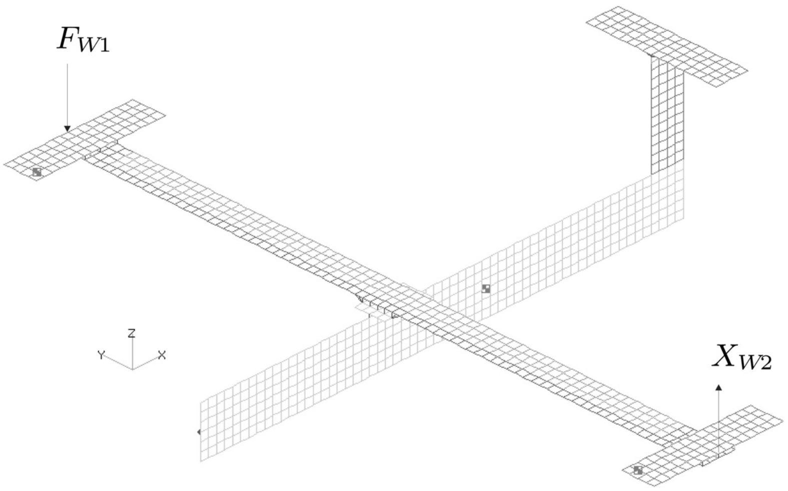


Fig. 2. Finite element model of the Garteur aircraft.

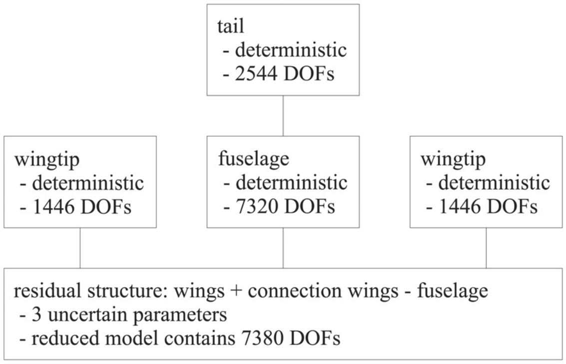


Fig. 3. Overview of the substructured Garteur aircraft.

reduced with the CMS method are investigated. In the substructured model, the tail, fuselage and both wingtips are modelled as deterministic components. The residual structure consists of the wings and the connection between wings and fuselage. Hence, all uncertain parameters affect only this residual structure. Fig. 3 gives a graphical overview of the substructuring of the model and indicates the number of DOFs of all substructures and the assembled reduced model. For the reduced model, a reduction of computation time of 20% is achieved. This reduction affects every goal function evaluation in the optimisation step; therefore, the same proportional time gain is achieved for the total IFEM procedure.

Fig. 4 presents the results of the interval FRF analysis compared with the results of a Monte Carlo Simulation with 100 samples. The upper and lower FRF bound calculated with the IFEM gives a narrow circumscription of the Monte Carlo results for the entire frequency domain. The use of the CMS method gives no loss of accuracy as the interval FRF results of the full and the reduced model coincide. Table 1 lists the eigenfrequency ranges for both models. Both the eigenfrequencies of the nominal case as the eigenfrequency intervals are predicted accurately with the reduced model.

4. Conclusions

This paper uses the IFEM for the dynamic analysis of structures with uncertain parameters. Eigenfrequency intervals and envelope FRFs are calculated for the Garteur benchmark aircraft, on one hand with a full model and on the other hand with a model reduced by the CMS method. The IFEM proves to be powerful and reliable: a conservative approximation of the upper and lower bound of an FRF is calculated in a single run. The use of a substructuring method enhances the efficiency of the IFEM, without compromising the accuracy of the interval results.

In the presented case study, uncertainties are located only in the model of the non-reduced residual structure. Further model reduction and hence reduction on the

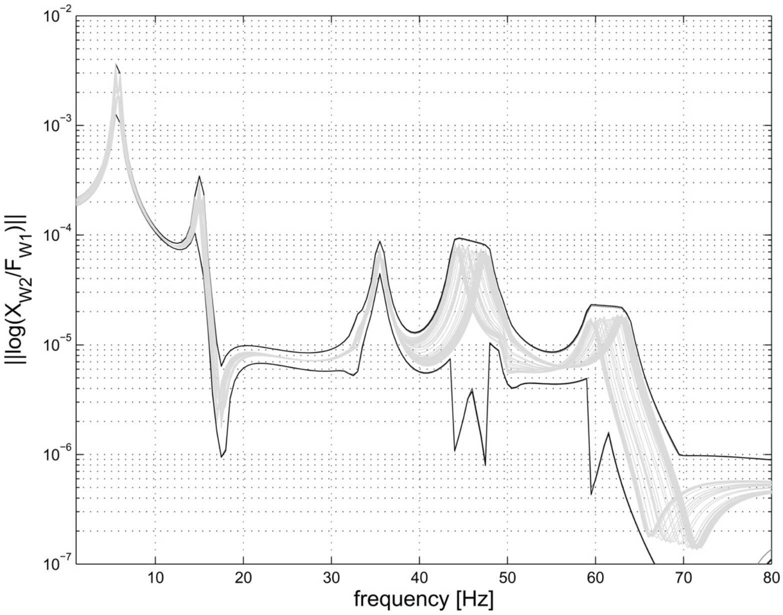


Fig. 4. Amplitude of the damped interval FRF of the Garteur aircraft model, compared with 100 Monte Carlo samples.

Table 1
Nominal eigenfrequency values (Hz), eigenfrequency intervals (Hz) and procentual width of the intervals (%) of the first 14 modes of the Garteur aircraft model, for the full model and the reduced model

	Full model			Reduced model		
Mode no.	Nominal value	Frequency interval	Interval width (%)	Nominal value	Frequency interval	Interval width (%)
1	5.8095	5.4886–5.9137	7.75	5.8095	5.4885–5.9137	7.75
2	15.203	14.727–15.365	4.33	15.203	14.727–15.365	4.33
3	33.077	32.417–33.379	2.97	33.102	32.442–33.404	2.97
4	33.204	32.485–33.532	3.22	33.232	32.512–33.560	3.22
5	35.609	35.263–35.766	1.43	35.614	35.267–35.772	1.43
6	46.640	43.926–47.804	8.83	46.654	43.937–47.818	8.83
7	49.821	49.269–50.007	1.50	49.820	49.269–50.008	1.50
8	54.029	53.482–54.347	1.62	54.025	53.476–54.345	1.63
9	62.358	59.361–63.617	7.17	62.387	59.382–63.647	7.18
10	67.595	67.583–67.603	0.03	67.595	67.583–67.603	0.03
11	100.24	100.10–100.33	0.23	100.24	100.10–100.33	0.23
12	128.81	126.26–129.58	2.63	128.89	126.37–129.64	2.59
13	137.44	129.50–141.12	8.97	137.77	129.77–141.47	9.02
14	150.85	143.89–153.91	6.97	151.04	143.99–154.14	7.05

calculation time can be obtained by permitting uncertainties in the component models. This implies the need for a concept for the description of uncertainty in the reduced matrices that represent a component that is subjected to uncertain parameters. In the future, larger structures also will be investigated.

Notes

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References

[1] Moens D. A non-probabilistic finite element approach for structural dynamic analysis with uncertain parameters. PhD thesis, Katholieke Universiteit Leuven, Departement Werktuigkunde, Leuven, 2002.

[2] Moens D, Vandepitte D. An interval finite element approach for the calculation of envelope frequency response functions. Int J Num Meth Engng 2004;61:2480–2507.

[3] Craig RR, Bampton MCC. Coupling of substructures for dynamic analyses. AIAA J 1968;6:1313–1319.

[4] MSC. Nastran Superelement User's Guide. The MacNeal-Schwendler Corporation, Los Angeles, CA, 2001.

[5] Degener M, Hermes M. Ground vibration test and finite element analysis of the GARTEUR SM-AG19 testbed. Technical report IB 232–96 J 08, DLR – German Aerospace Research Establishment, Institute for Aeroelasticity, October 1996.