Using Singular Vectors and Rossby Waves Interaction to Understand Optimal Growth in Shear Flows

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Shear flows are inherently non-normal dynamical systems in the sense that their linearized eigen functions are not orthonormal. As a result, a superposition of the eigenfunctions might yield transient growth even if all their eigenvalues are negative. Hence, although the linearized system is asymptotically stable it can exhibit large growth in final time and become nonlinearly unstable.Consequently, the linearized stable asymptotic limit becomes irrelevant.

Furthermore, in non-normal unstable linear systems, the maximal possible growth is always larger than the growth given by the largest eigenvalue (the most unstable normal mode).Thus, in order to obtain the optimal growth, that is the largest possible growth the system extracts for a given target time, one needs to apply a Singular Value Decomposition (SVD) of the propagator matrix, rather than an eigen value decomposition. The first singular value gives the maximal growth where the structure of the optimally growing perturbation is given by the singular vectors.

Here, we will present the SVD formulation and its application for non-normal dynamical systems. Next we will use the Rossby Wave perspective to find the vorticity perturbation building blocks and will describe the optimal evolution in terms of constructive interaction between those waves.